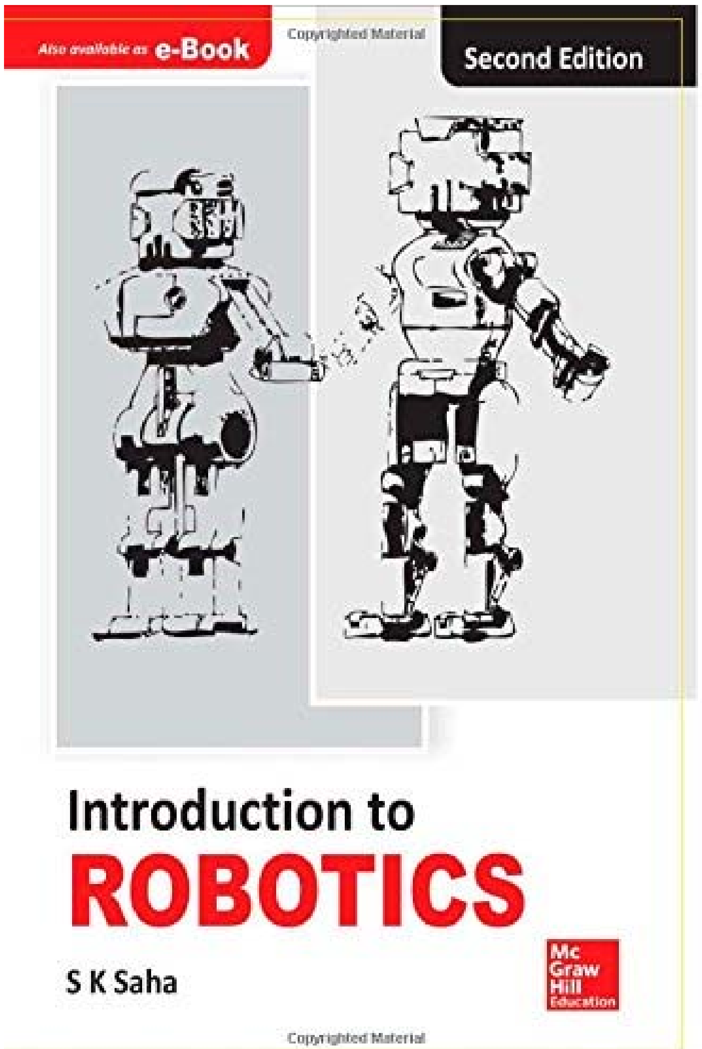
Lecture 4
Kinematics and Statics
by
S.K. Saha
Aug. 24’16 (W)@JRL301 (Robotics Tech.)



Chapters
6 and 7

Recap

- DH Paramters
- Forward kinematics
- Inverse kinematics

Numerical Example

- An RRR planar arm (Example 6.15). Input

$\mathbf{T} \equiv$ [Rotation Matrix | Origin of end-effector frame]

4.23

1.86

0

$$\begin{matrix} 0 & 0 & 0 & 1 \end{matrix}$$

where $\phi = 60^{o}$, and $a_1 = a_2 = 2$ units, and $a_3 = 1$ unit.

Do it yourself → Verify using RoboAnalyzer

Using eqs. (6.13b-c), $c_2 = 0.866$, and $s_2 = 0.5$,

$\theta_2 = 30^o$

Next, from eqs. (6.16a-b), $s_1 = 0$, and $c_1 = 0.866$.

$\theta_1 = 0^o$.

Finally, from eq. (6.17) , $\theta_3 = 30^o$.

MATLAB program

Therefore $\theta_1 = 0^o\ \theta_2 = 30^o$, and $\theta_3 = 30$ …(6.30b)

The positive values of s_2 was used in evaluating $\theta_2 = 30^o$.

The use of negative value would result in :

$\theta_1 = 30^o\ \theta_2 = -30^o$, and $\theta_3 = 60^o$ …(6.30c)

Extra Reading: Watch

- Forward and Inverse Kinematics: Watch 3/3 of IGNOU Lectures [29min]

https://www.youtube.com/watch?v=duKD8cvtBTI

- For more clarity: Watch 12 of Addis Ababa Lectures [77 min]

[https://www.youtube.com/watch?v=NXWzk1toze4

- Robotics (13 of Addis Ababa Lectures): Inverse Kinematics [82 min]

https://www.youtube.com/watch?v=ulP3YiJLiEM

Velocity Analysis

Jacobian maps joint rates into end-effector's velocities. It depends on the manipulator configuration.

$$\text{twist of end-effector}: \mathbf{t}_e \equiv \begin{bmatrix} \boldsymbol{\omega}_e \\ \mathbf{v}_e \end{bmatrix}; \text{ Joint rates}: \dot{\boldsymbol{\theta}} = \begin{bmatrix} \dot{\theta}_1 \\ \vdots \\ \dot{\theta}_n \end{bmatrix}$$

$$\mathbf{t}_\mathbf{e} = \mathbf{J}\dot{\boldsymbol{\theta}} \qquad \text{where } \mathbf{J} = \begin{bmatrix} \mathbf{j}_1 & \mathbf{j}_2 & \cdots & \mathbf{j}_n \end{bmatrix} \text{ and}$$

$$\mathbf{J} = \begin{bmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \cdots & \mathbf{e}_\mathbf{n} \\ \mathbf{e}_1 \times \mathbf{a}_{1e} & \mathbf{e}_2 \times \mathbf{a}_{2e} & \cdots & \mathbf{e}_\mathrm{n} \times \mathbf{a}_{ne} \end{bmatrix} \qquad \text{. . (6.86)}$$

$$\mathbf{j}_\mathrm{i} \equiv \begin{bmatrix} \mathbf{e}_\mathrm{i} \\ \mathbf{e}_\mathrm{i} \times \mathbf{a}_\mathrm{ie} \end{bmatrix}, \text{ if Joint } i \text{ is revolute} \quad \mathbf{j}_i \equiv \begin{bmatrix} \mathbf{0} \\ \mathbf{e}_i \times \mathbf{a}_{ie} \end{bmatrix}, \text{ if Joint } i \text{ is prismatic}$$

Jacobian of a 2-link Planar Arm

$$\mathbf{J} = \begin{bmatrix} \mathbf{e}_1 \times \mathbf{a}_{1e} & \mathbf{e}_2 \times \mathbf{a}_{2e} \end{bmatrix}$$

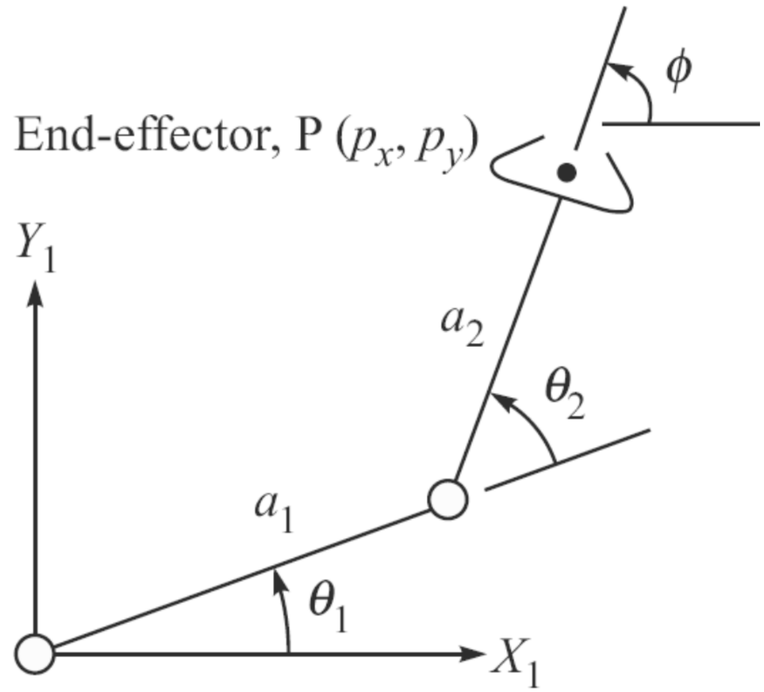
where $\mathbf{e}_1 \equiv \mathbf{e}_2 \equiv [0 \quad 0 \quad 1]^T$

$$\mathbf{a}_{1e} \equiv \mathbf{a}_1 + \mathbf{a}_2$$

$$\equiv [a_1 c_1 + a_2 c_{12} \quad a_1 s_1 + a_2 s_{12} \quad 0]^T$$

$$\mathbf{a}_{2e} \equiv \mathbf{a}_2$$

$$\equiv [a_2 c_{12} \quad a_2 s_{12} \quad 0]^T$$



Hence, $\mathbf{J} = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \end{bmatrix}$

Example: Singularity of 2-link RR Arm

$$\mathbf{J} \equiv \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \end{bmatrix}$$

$$\theta_2 = 0 \text{ or } \pi$$

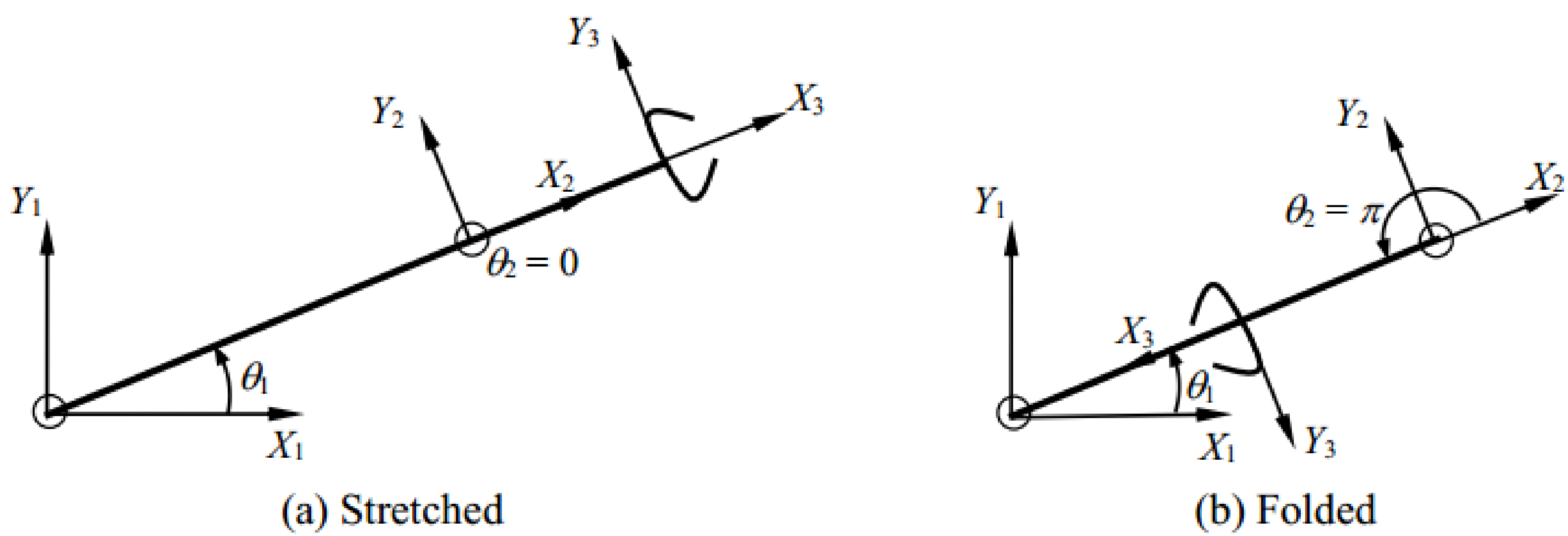


Figure 7.9 Singular configurations of a two-link planar arm

Chapter 7
Statics

Statics: Principle of Virtual Work

$$\mathbf{w}_e^T \delta\mathbf{x} = \boldsymbol{\tau}^T \delta\boldsymbol{\theta} \quad \dots (7.28)$$

- Relation between two virtual displacements (Can be derived from velocity expression)

$$\delta\mathbf{x} = \mathbf{J}\delta\boldsymbol{\theta} \quad \dots (7.29)$$

$$\mathbf{w}_e^T \mathbf{J}\delta\boldsymbol{\theta} = \boldsymbol{\tau}^T \delta\boldsymbol{\theta} \Rightarrow \mathbf{w}_e^T \mathbf{J} = \boldsymbol{\tau}^T \quad \dots (7.31)$$

$$\boxed{\boldsymbol{\tau} = \mathbf{J}^T \mathbf{w}_e} \quad \dots (7.32)$$

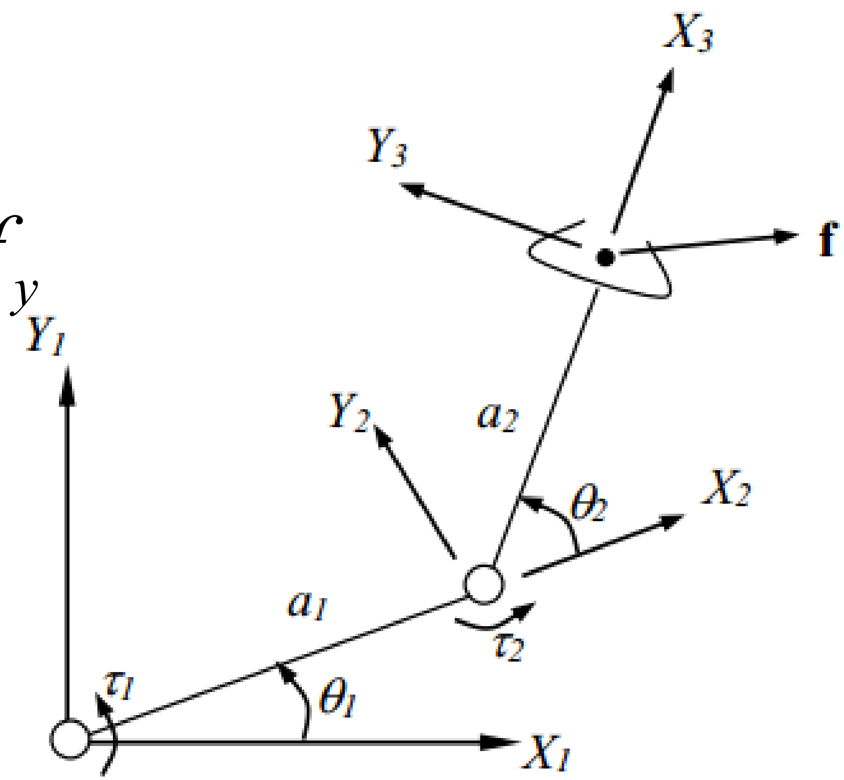
Example: 2-link RR Planar Arm

From FBD

$$\tau_1 = [\mathbf{e}_1]_1^T [\mathbf{n}_{01}]_1$$

$$= a_1 f_x s\theta_2 + (a_2 + a_1 c\theta_2) f_y$$

$$\tau_2 = [\mathbf{e}_2]_2^T [\mathbf{n}_{12}]_2 = a_2 f_y$$



$$\boldsymbol{\tau} = \mathbf{J}^T \mathbf{f}$$

$$\boldsymbol{\tau} \equiv \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \quad \mathbf{J}^T \equiv \begin{bmatrix} a_1 s\theta_2 & a_1 c\theta_2 + a_2 & 0 \\ 0 & a_2 & 0 \end{bmatrix} \quad \mathbf{f} \equiv \begin{bmatrix} f_x \\ f_y \\ 0 \end{bmatrix}$$

Two Jacobian Matrices

- From Statics

$$\mathbf{J} \equiv \begin{bmatrix} a_1 s\theta_2 & 0 \\ a_1 c\theta_2 + a_2 & a_2 \\ 0 & 0 \end{bmatrix}$$

- From Kinematics

$$\mathbf{J} \equiv \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \end{bmatrix}$$

Jacobian from Statics in Frame 1

$$[\mathbf{J}]_1 \equiv \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 \\ s\theta_2 & c\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_1 s\theta_2 & 0 \\ a_1 c\theta_2 + a_2 & a_2 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -a_1 s\theta_1 - a_2 s\theta_{12} & -a_2 s\theta_{12} \\ a_1 c\theta_1 + a_2 c\theta_{12} & a_2 c\theta_{12} \\ 0 & 0 \end{bmatrix} \quad \ldots (7.34)$$

- Without the last row, it is the same as the one from kinematics ← Should be!

Summary

- Velocity Jacobian
- Statics

THANK YOU

saha@mech.iitd.ac.in

http://sksaha.com