McGraw Hill Education

Lecture 5
Dynamics
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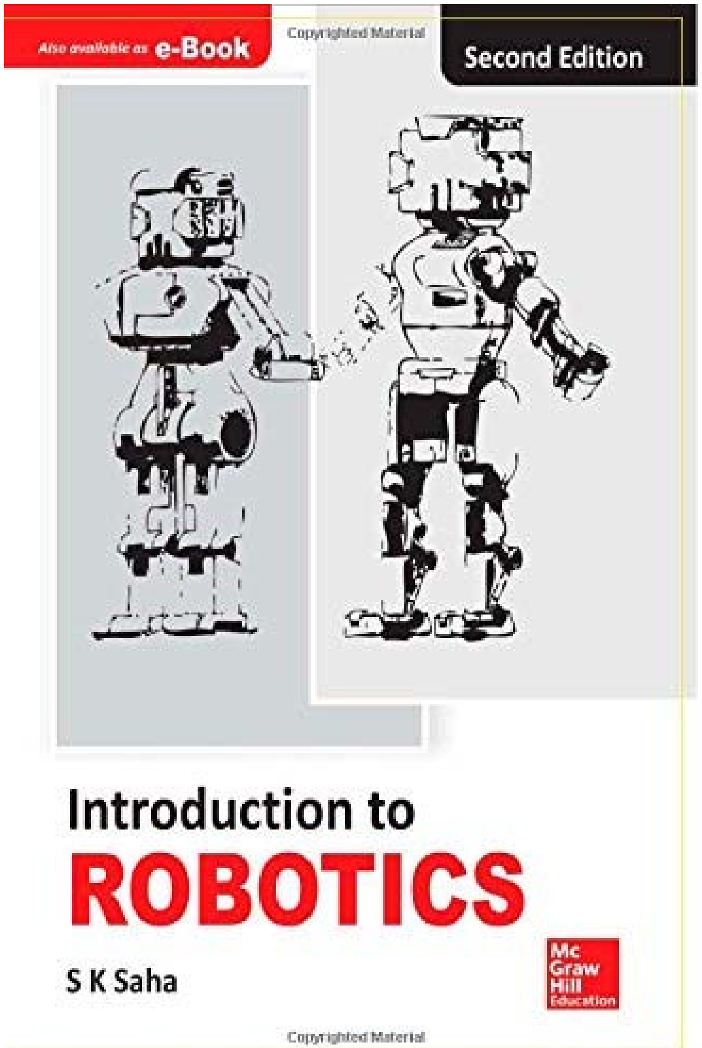
Aug. 31’16 (W)@JRL301 (Robotics Tech.)



Chapter 8

Recap

- Inverse kinematics
- Velocity analysis
- Jacobian
- Statics

Euler-Lagrange Formulation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = \phi_i$$

L (Lagrangian) = $T - U$;

T: Kinetic energy; U: Potential energy;

q_i: Generalized coordinate;

ϕ_i : Generalized force.

Generalized Coordinates

- Coordinates that specify the configuration (position and orientation) → generalized coordinates

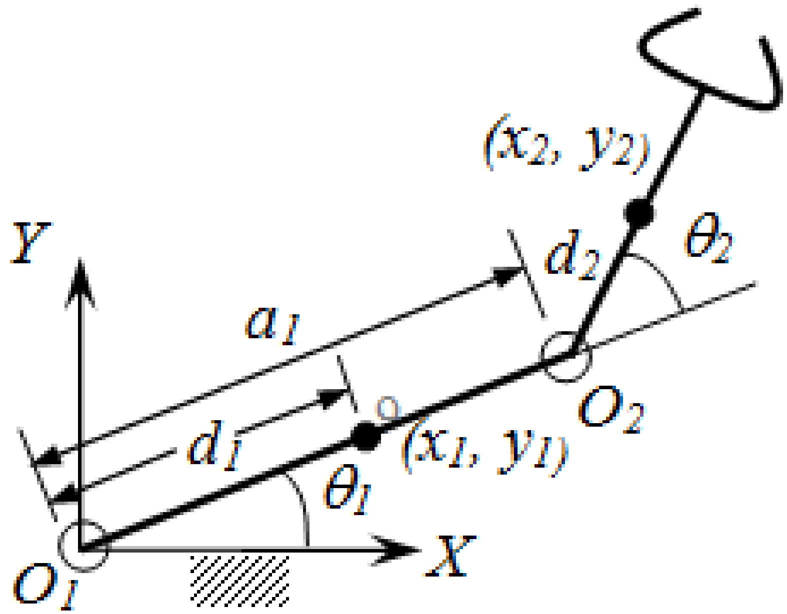


Figure 8.5 A two-link robot arm

Kinetic and Potential Energies

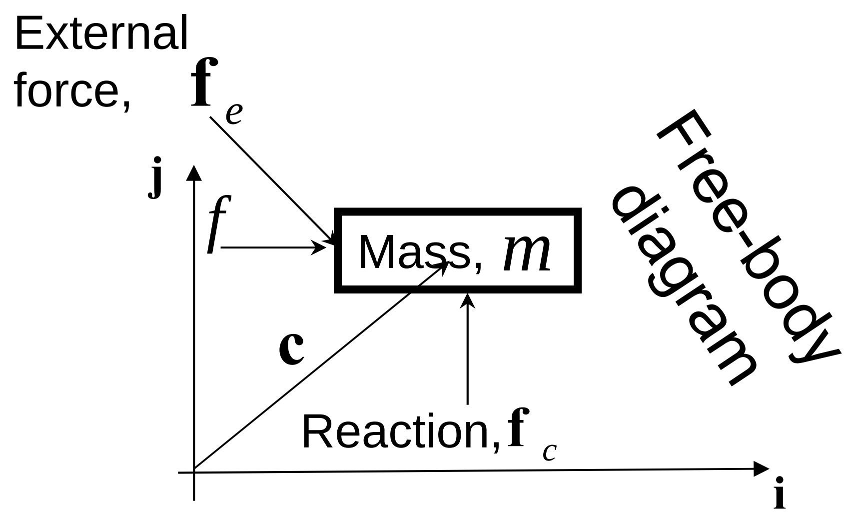
- Kinetic Energy

$$T = \sum_{i=1}^{n} T_i = \sum_{i=1}^{n} \frac{1}{2}\left(m_i \dot{\mathbf{c}}_i^T \dot{\mathbf{c}}_i + \boldsymbol{\omega}_i^T \mathbf{I}_i \boldsymbol{\omega}_i \right)$$

- Potential Energy

$$U = -\sum_{i=1}^{n} m_i \mathbf{c}_i^T \mathbf{g}$$

Euler-Lagrange Equation



Kinetic energy $T = \frac{1}{2} m \dot{\mathbf{c}}^T \dot{\mathbf{c}}; \; U = 0$

Velocity constraint: $\dot{\mathbf{c}} = \dot{x}\mathbf{i}$; $L (= T - U) = \frac{1}{2} m \dot{x}^2$

$$\frac{\partial L}{\partial \dot{x}} = m\dot{x}; \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = m\ddot{x}; \quad \frac{\partial L}{\partial x} = 0$$

Euler-Lagrange: $\boxed{m\ddot{x} = f}$

Example: One-DOF Arm (EL)

$T \equiv$ Please write!

$$U = mg\left(\frac{a}{2} - \frac{a}{2}c\theta\right)$$

$$L = T - U \equiv \qquad - mg\frac{a}{2}(1 - c\theta)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \qquad \frac{\partial L}{\partial \theta} =$$

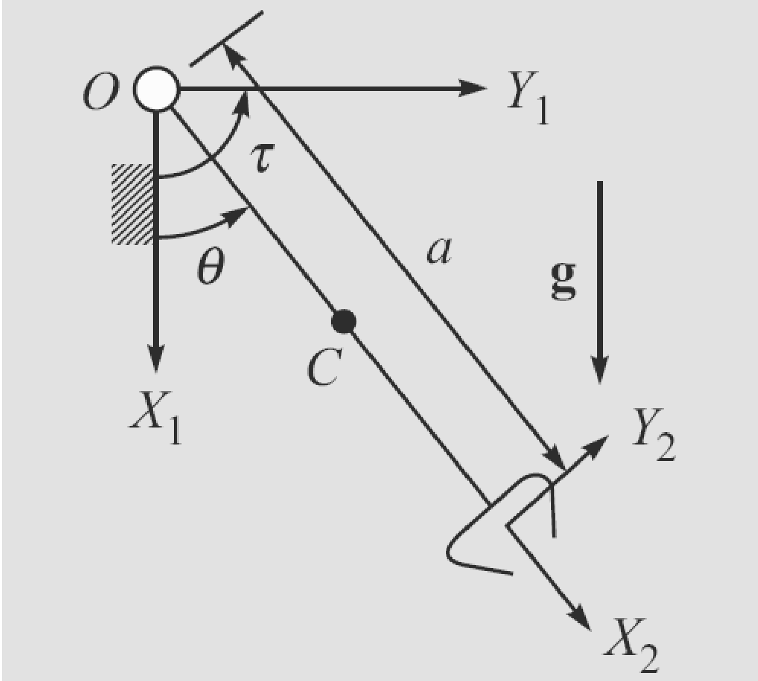


Fig. 8.7 One-link arm

Simulation of One-link Arm

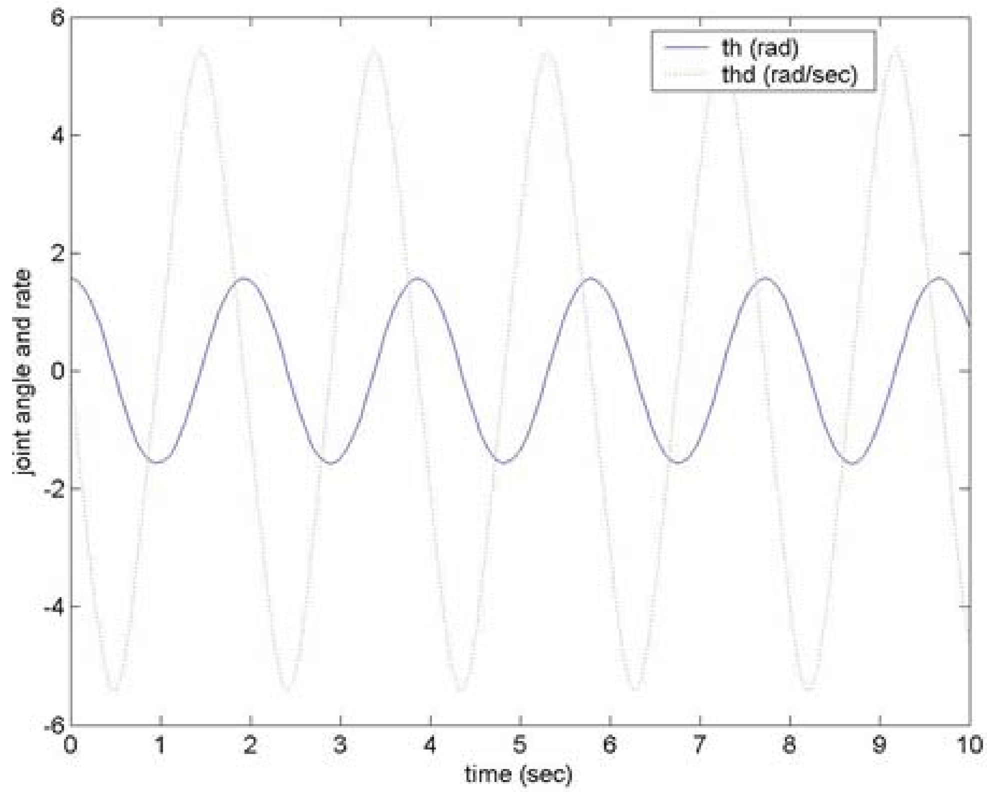


Figure 8.19 Simulation results of one-link arm under gravity

RoboAnalyzer

Simulation of One-link Arm using MATLAB

$$\ddot{\theta} = \frac{2}{ma^2}(\tau - \frac{1}{2} mga \sin\theta)$$

Hence, the state-space form is given by

$$\dot{y}_1 = y_2$$

$$\dot{y}_2 = \frac{2}{ma^2}(\tau - \frac{1}{2} mga \sin\theta)$$

```
%For one-link arm
function ydot =ch8fdyn1(t,y);
m = 1; a = 1; g = 9.81; tau=0;
iner = m*a*a/3; grav = m*g*a/2;
ydot=[y(2);(tau-grav*sin(y(1)))/iner];
```

(a) Program for state-space form

```
%For one link arm
tspan=[0 10]; y0=[pi/2; 0];
[t,y]=ode45('ch8fdyn1',tspan,y0)
```

(b) Program to integrate numerically

Figure 8.18 Simulation of one-link arm under gravity only

Summary

- Euler-Lagrange equation was shown
  - Generalized coordinates, generalized forces were defined
- Demonstration with RoboAnalyzer and MATLAB

THANK YOU

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