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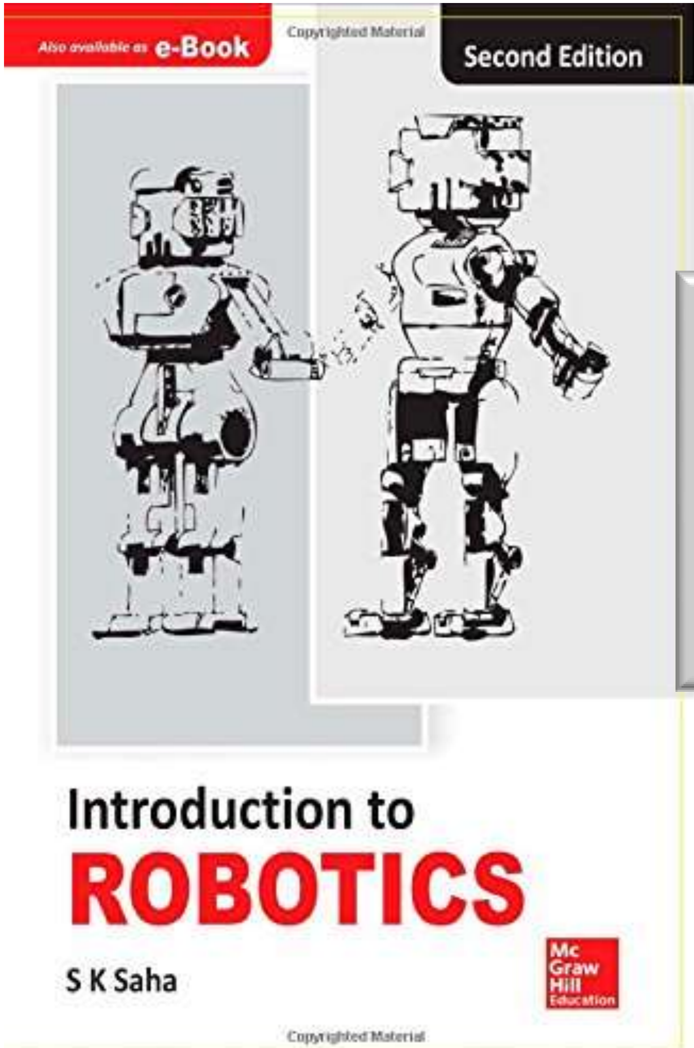

Lecture 6 (SIT Sem. Rm.)

Mobile Robots

by

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Sep. 05, 2016 (M)@JRL301(Robotics Tech.)

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Recap

- Euler-Lagrange equation was shown
  - Generalized coordinates, generalized forces were defined
- Demonstration with RoboAnalyzer and MATLAB

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Mobile Robots

- Non-holonomic systems
  - Necessary and sufficient no. of variables defining a pose exceeds the number of actuators
- Holonomic
  - Necessary and sufficient no. of variables defining a pose is same as the no. independent actuators

[Ref: Dynamics and Design of Nonholonomic Robotic Mechanical Systems, Ph. D thesis, McGill Univ., Canada, 1991]

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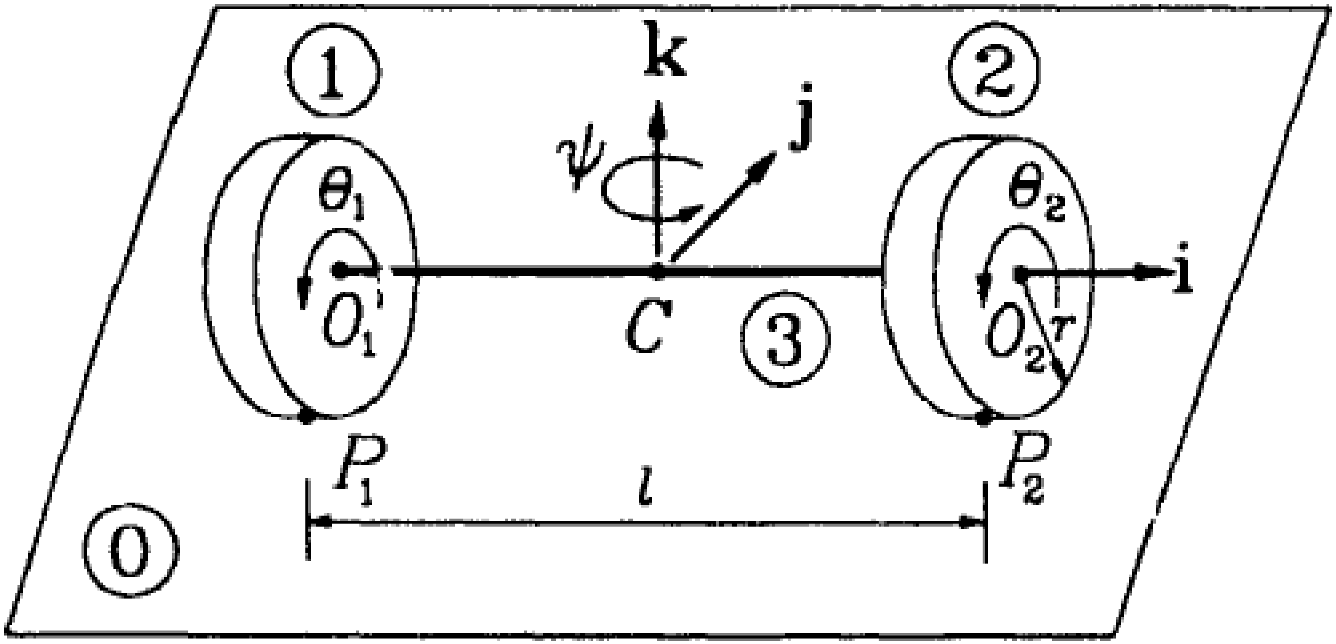
Two-wheeled System



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Kinematic and Dynamic Models

(Handout)

$$\mathbf{t}_C = \mathbf{T}_C \dot{\boldsymbol{\theta}}_l$$

$$\mathbf{t}_C \equiv [\boldsymbol{\omega}^T, \quad \mathbf{c}^T]^T \qquad \dot{\boldsymbol{\theta}}_l \equiv [\dot{\theta}_1, \quad \dot{\theta}_2]^T \qquad \mathbf{T}_C = \frac{\eta}{2}\begin{bmatrix} 2\mathbf{k} & -2\mathbf{k} \\ -l\mathbf{j} & -l\mathbf{j} \end{bmatrix}$$

$$\eta = r/l$$

$$\mathbf{I}\ddot{\boldsymbol{\theta}}_l = \boldsymbol{\tau}$$

$$\mathbf{I} = \frac{mr^2}{2}\begin{bmatrix} 3+\eta^2 & -\eta^2 \\ -\eta^2 & 3+\eta^2 \end{bmatrix}. \qquad \boldsymbol{\tau} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

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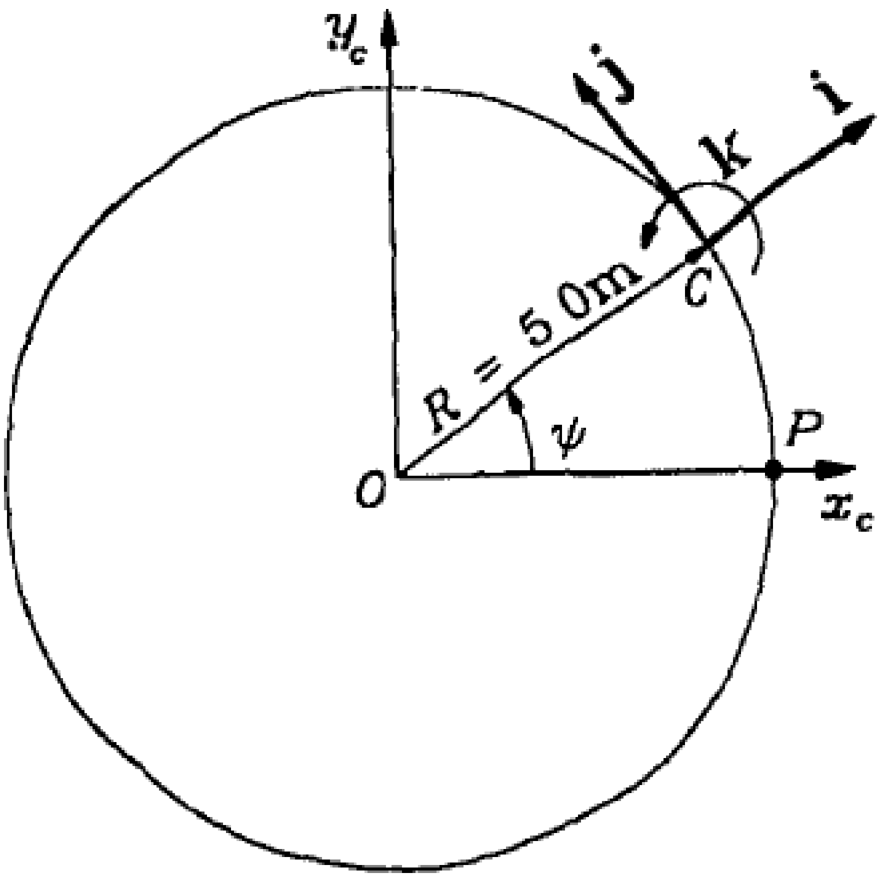
Circular Path



Time: 60 sec
Radius of wheel, *r*
Mass of wheels, *m*
Length, *l*

$r = 0.05$ m,

$l = 0.4$ m,

$m = 2.0$ kg

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Why Motion Planning?

- Just like a person to decide a route from one place to another, a robot's path needs to be decided through motion planning.
- Goal of motion planning is to generate a function according to which a robot will move.
- Function generation depends on the robot tasks

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Motion (or Trajectory) Planning

- In joint space, i.e., in terms of joint positions, velocities and accelerations, or
- Cartesian (or operational) space, i.e., in terms of the end-effector positions, orientations, and their time derivatives. This is preferred as it allows a natural description of the task the robot has to perform.
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Role of Inverse Kinematics

- However, the control action on the robot is carried in the joint space.
- A suitable inverse kinematics algorithm is to be used to reconstruct the time sequence of joint variables corresponding to the above sequence in the Cartesian space.

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Joint Space Planning

- Suppose, for any joint, the initial angle is given at time, t = 0, and the final angle is known at time t = tf, i.e.,

$$\theta(0) = \theta_o \,; \text{and} \;\; \theta(t_f) = \theta_f$$

- Additionally, to start and stop the EE according to some desired velocity and acceleration,

$$\dot{\theta}(0) = \dot{\theta}_o \;;\; \dot{\theta}(t_f) = \dot{\theta}_f$$

$$\ddot{\theta}(0) = \ddot{\theta}_o \;;\; \ddot{\theta}(t_f) = \ddot{\theta}_f$$

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Joint Space Planning (contd….)

- Straightforward approach could be to join the initial and final joint angles as straight line, i.e.,

$$\theta(t) = a_o + a_1 t \quad \text{where} \quad a_o \equiv \theta(t_0) \quad a_1 \equiv \frac{\theta(t_f) - \theta(t_0)}{t_f - t_0}$$

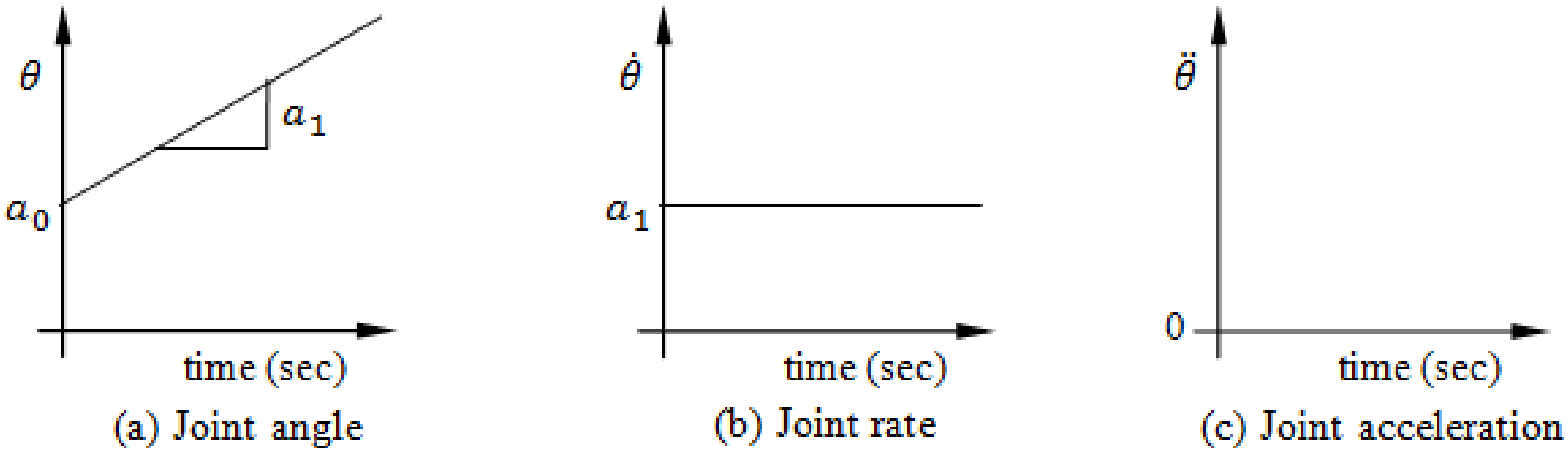

Figure 12.1 A straight-line trajectory

Above trajectory not desired due to its discontinuities in joint rates.

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Cubic Polynomials

- Both the velocity and acceleration levels continuity need to be maintained

$$\theta(t) = a_o + a_1 t + a_2 t^2 + a_3 t^3$$ … (12.2a)

velocity and accelerations conditions at time t_0 and t_f

$$\theta(t_0) = \theta_{\mathbf{o}} \; ; \; \theta(t_f) = \theta_f \qquad \dot{\theta}(t_0) = \dot{\theta}_{\mathbf{o}} \; ; \; \dot{\theta}(t_f) = \dot{\theta}_f$$

Differentiating Eq. (12.2a) for velocity and acceleration and arranging to find the coefficients a_0, a_1, a_2, and a_3 with linear algebraic equations of form

$$\mathbf{Ax} = \mathbf{b}$$

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Cubic Polynomials (contd….)

where

$$\mathbf{A} \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & t_f & t_f^2 & t_f^3 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 2t_f & 3t_f^2 \end{bmatrix} \quad \mathbf{x} \equiv \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} \quad \mathbf{b} \equiv \begin{bmatrix} \theta_0 \\ \theta_f \\ \dot{\theta}_0 \\ \dot{\theta}_f \end{bmatrix}$$

Solution to the set of linear algebraic equations

$$a_o = \theta_o \;\; ; \;\; a_1 = \dot{\theta}_o$$

$$a_2 = \frac{1}{t_f^2}[3(\theta_f - \theta_o) - (2\dot{\theta}_o + \dot{\theta}_f)t_f]$$

$$a_3 = \frac{1}{t_f^3}[2(\theta_o - \theta_f) + (\dot{\theta}_o + \dot{\theta}_f)t_f]$$

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Quintic Polynomial

- Polynomials of time *t* are chosen due to their ease of computation and simplicity of expression.
- Lowest order that can satisfy all the above six conditions is five, i.e.,

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

$$\dot{\theta}(t) = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + 5a_5 t^4$$

$$\ddot{\theta}(t) = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + 5a_5 t^4$$

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Coefficients for Quintic Polynomial

- Five coefficients are:

$$a_0 = \theta_o \;;\; a_1 = \dot{\theta}_o \;;\; a_2 = \frac{1}{2}\ddot{\theta}_o$$

$$a_3 = \frac{1}{2t_f^3}\left[20(\theta_f - \theta_o) - 8(\dot{\theta}_f + 12\dot{\theta}_o)t_f - (3\ddot{\theta}_o - \ddot{\theta}_f)t_f^2\right]$$

$$a_4 = \frac{1}{2t_f^4}\left[30(\theta_o - \theta_f) + (14\dot{\theta}_f + 16\dot{\theta}_o)t_f + (3\ddot{\theta}_o - 2\ddot{\theta}_f)t_f^2\right]$$

$$a_5 = \frac{1}{2t_f^5}\left[12(\theta_f - \theta_o) - 6(\dot{\theta}_f + \dot{\theta}_o)t_f - (\ddot{\theta}_o - \ddot{\theta}_f)t_f^2\right]$$

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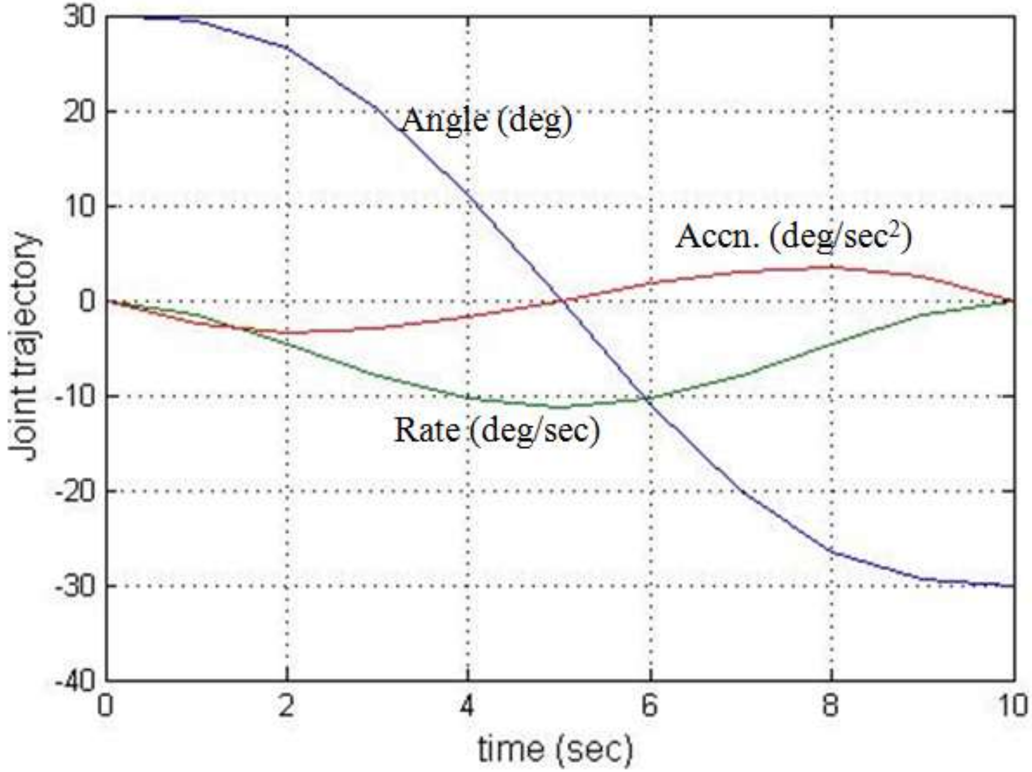
Quintic Polynomials

Quintic trajectory is shown below:



(a) Trajectory

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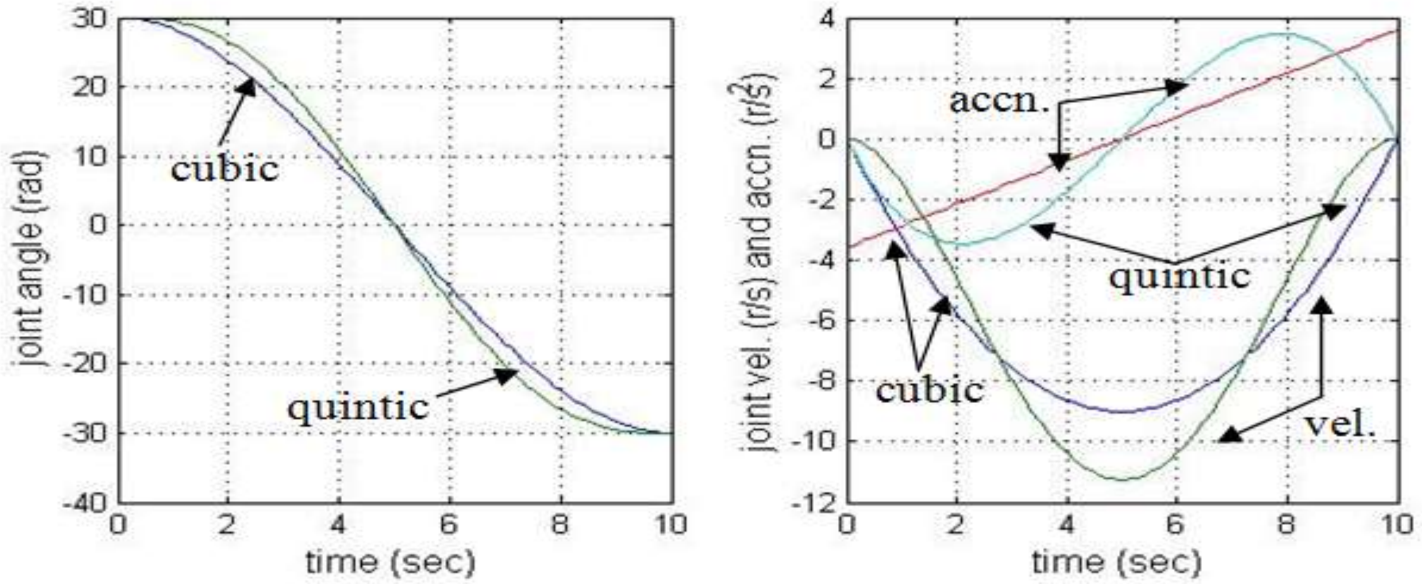
Comparison of a Cubic vs. Quintic Polynomials



(a) Joint angles (b) Joint velocities and accelerations

Figure 12.6 Comparison of cubic and quntic polynomials

Note: Quintic polynomial has no discontinuity in acceleration at the ends of the trajectory. It shoots in the middle of all three plots, i.e., angle, velocities, and accelerations.

As the order of polynomial increases such shooting up is more and more prominent.

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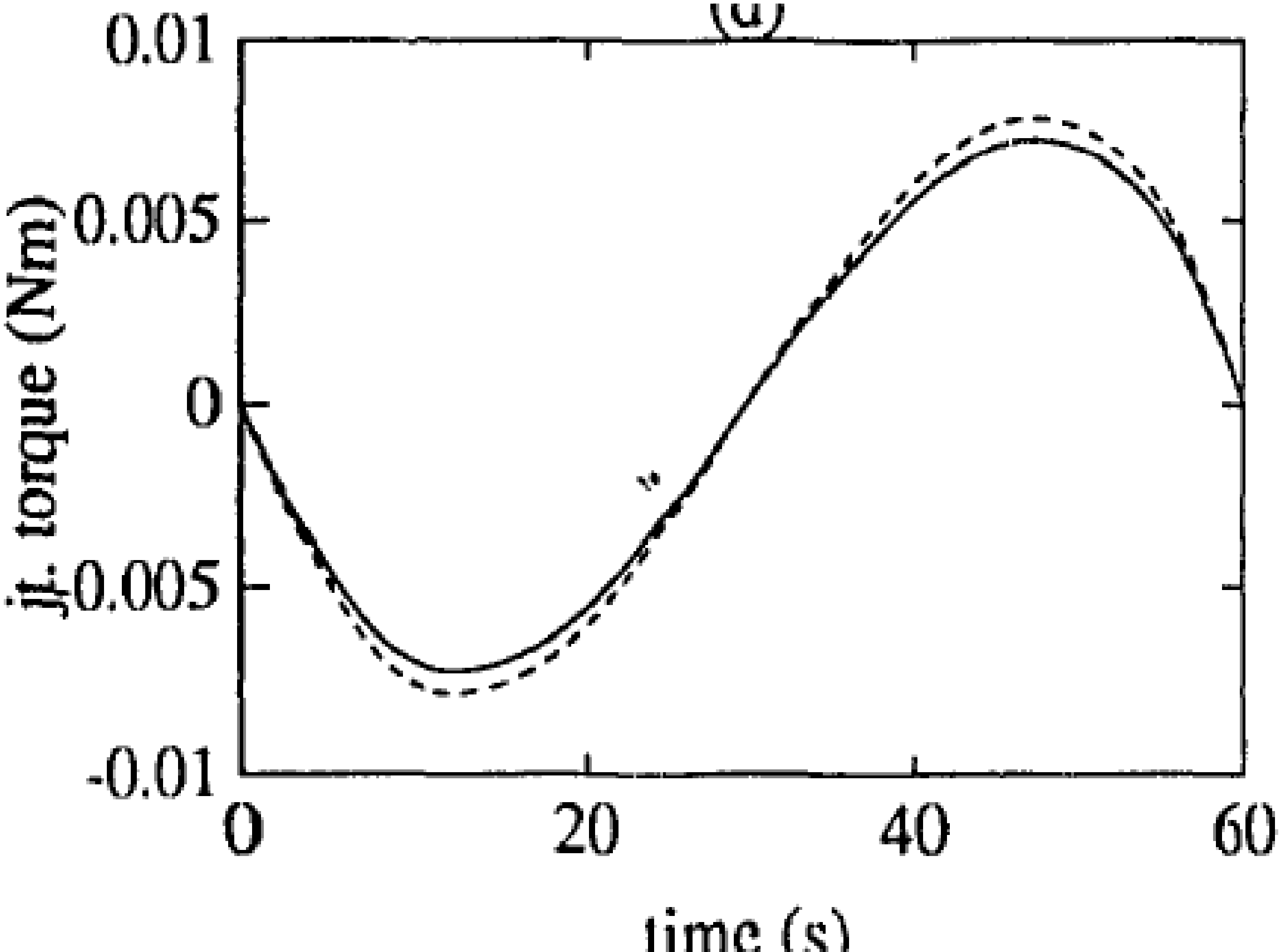
Joint Torques (- 1; .. 2)



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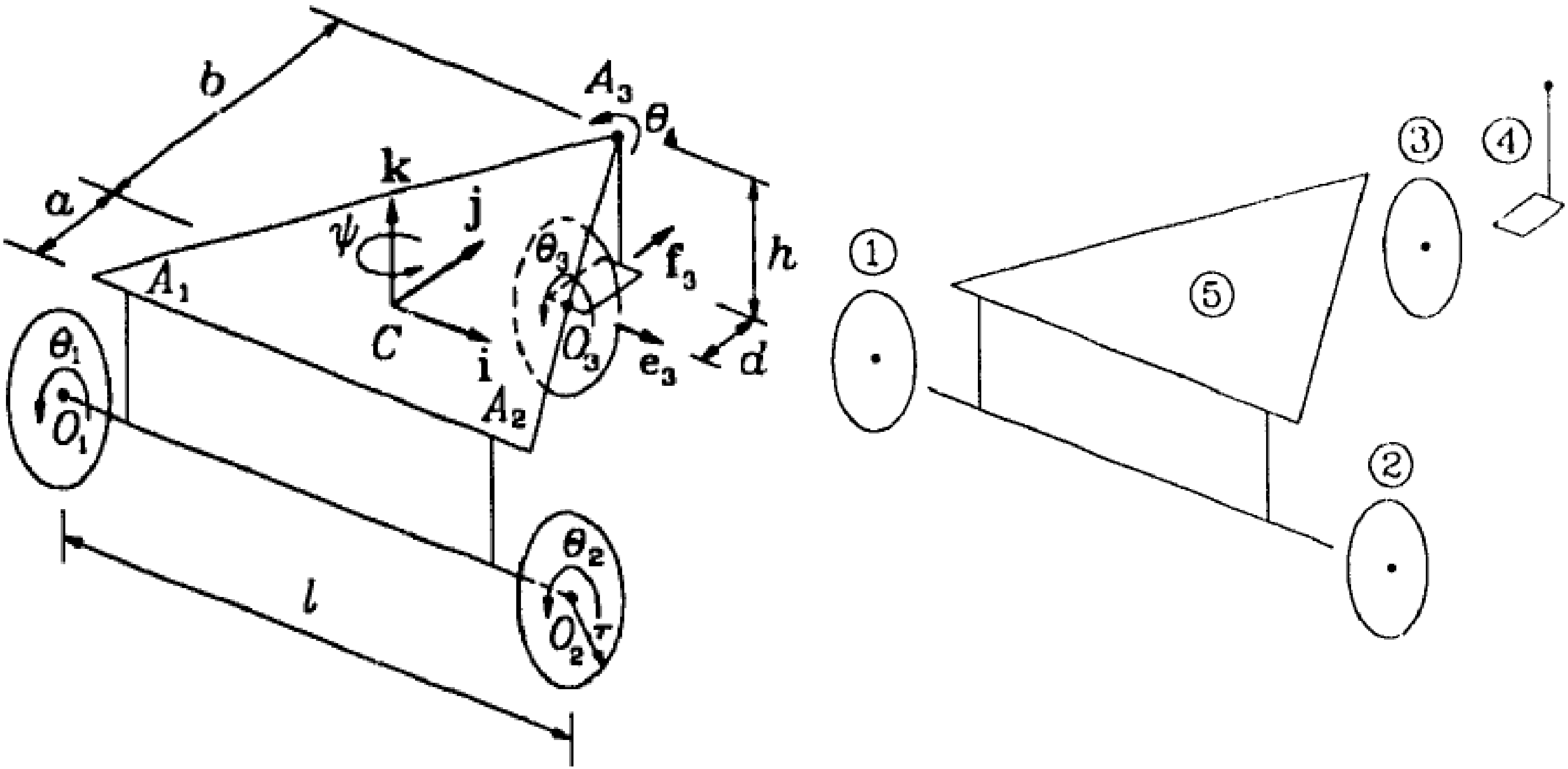
Three Wheeled 2-DOF AGV



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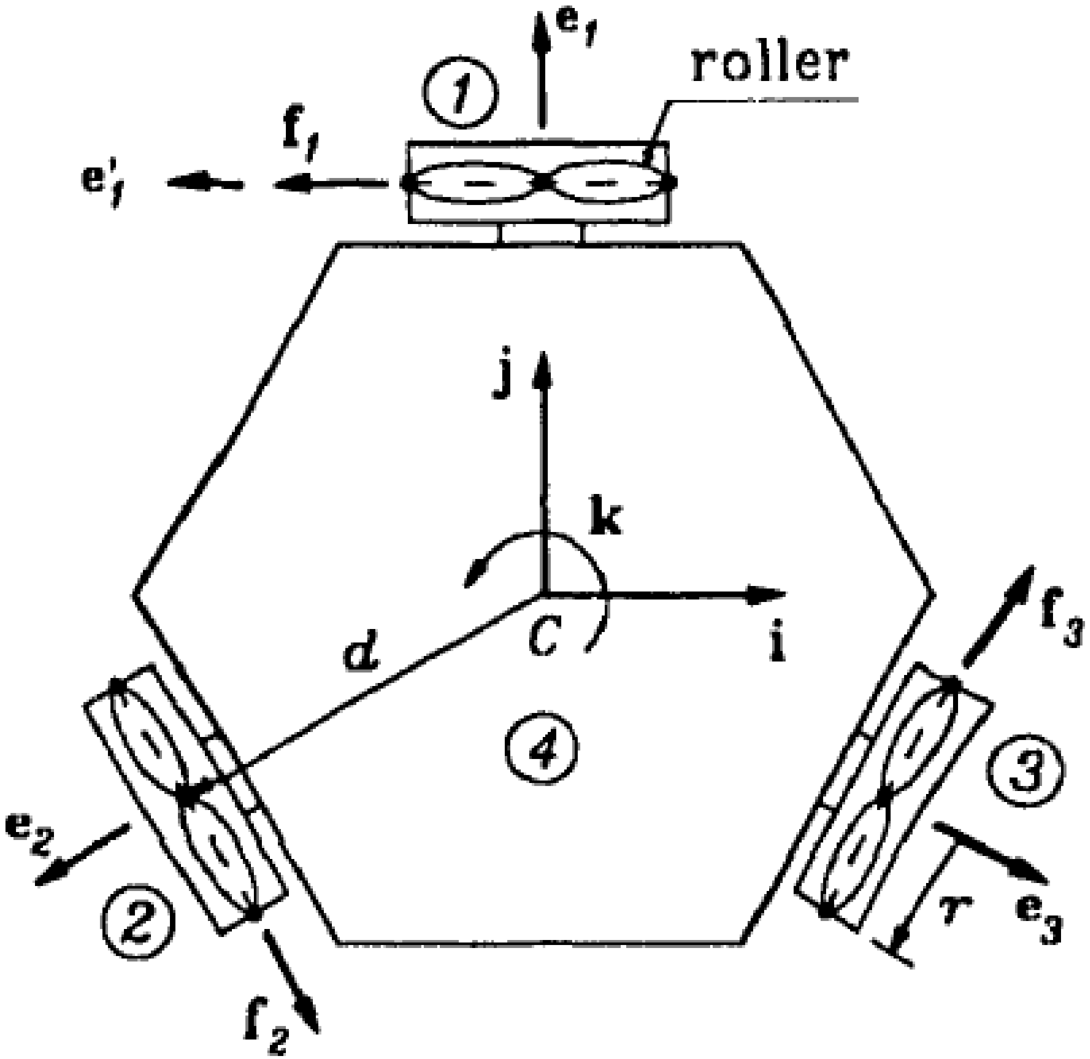
Three-DOF 3-Wheeled Mobile Robot



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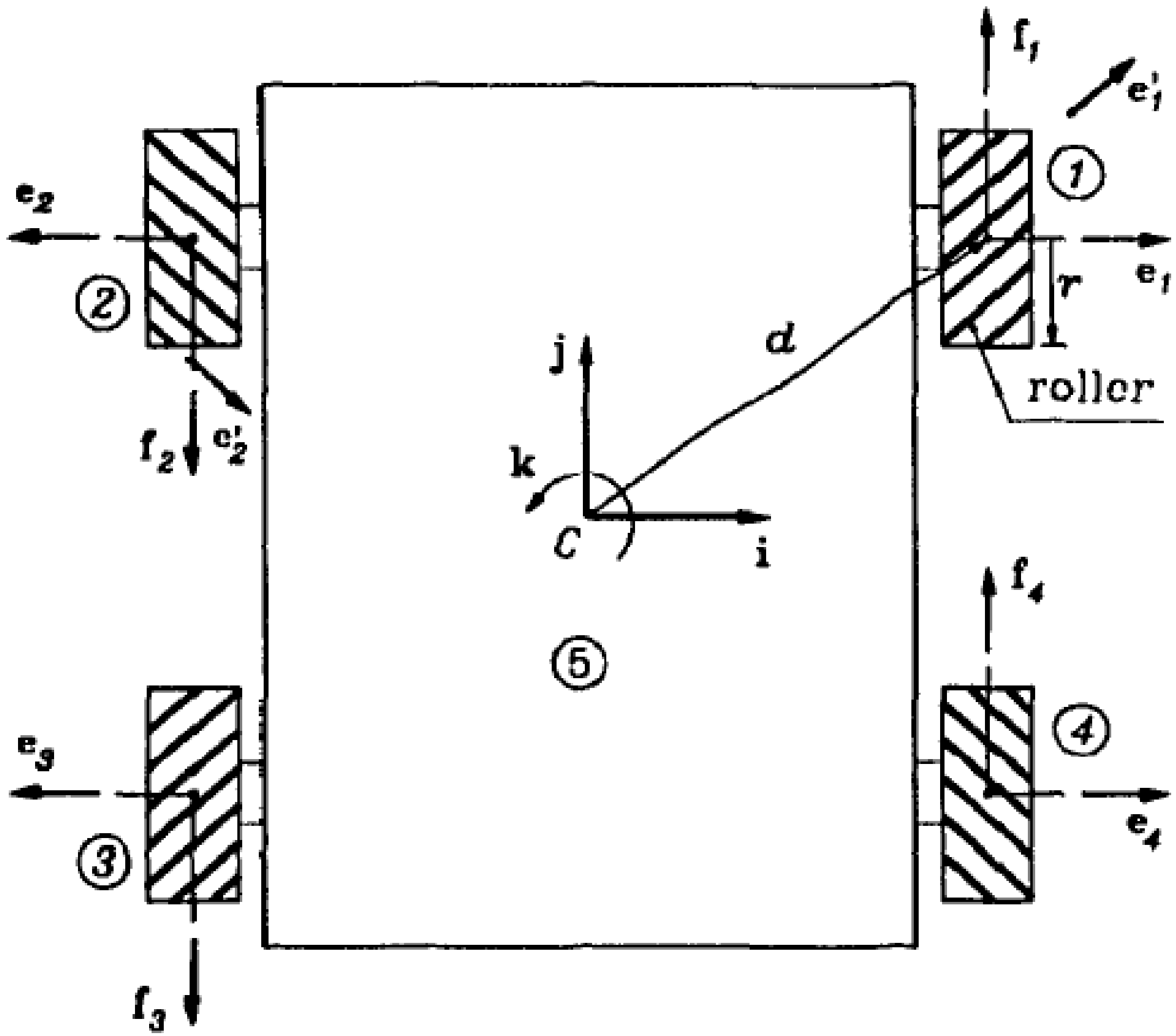
Three-DOF 4-Wheeled



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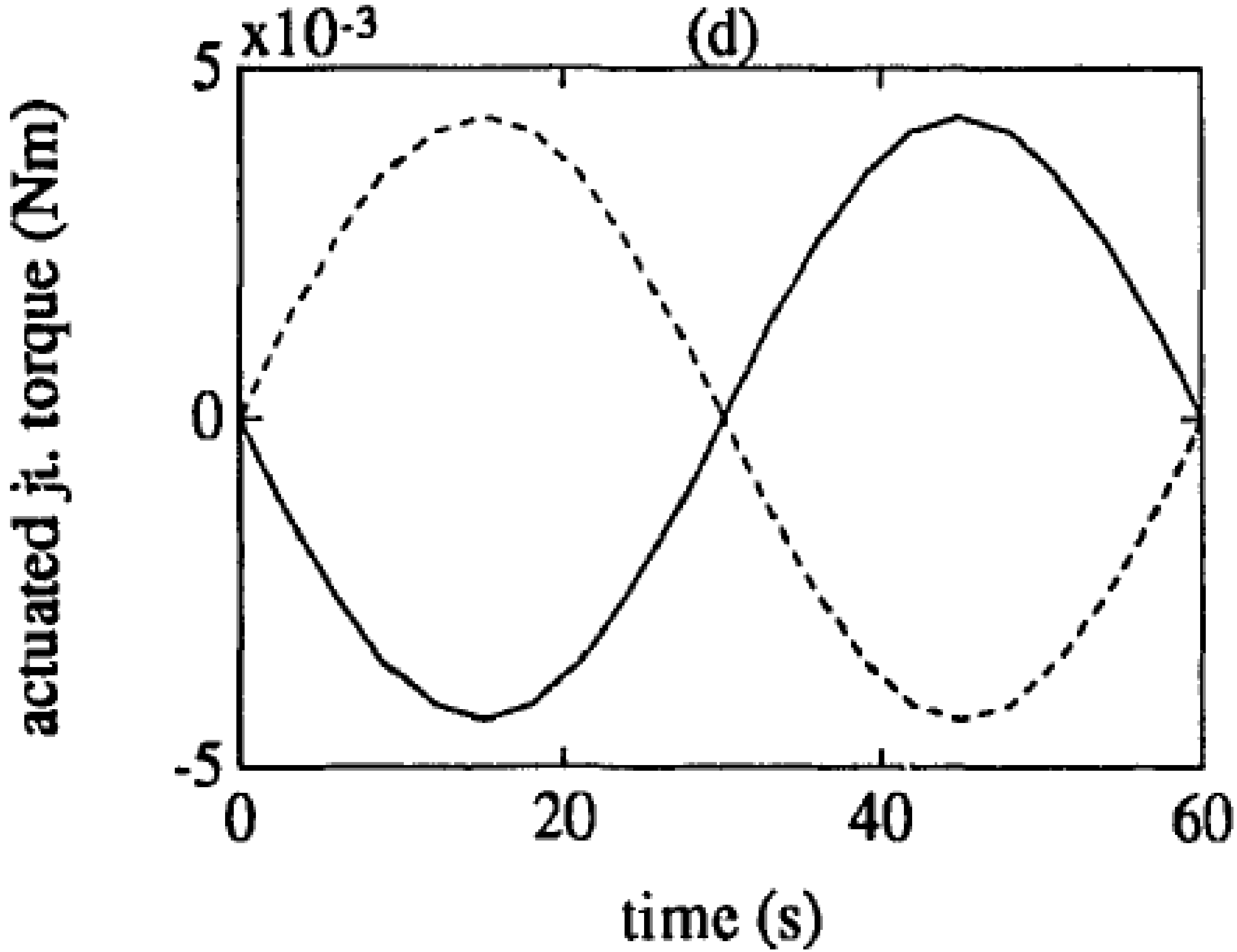
Joint Torques@**j** (- 1&4; .. 2&3)



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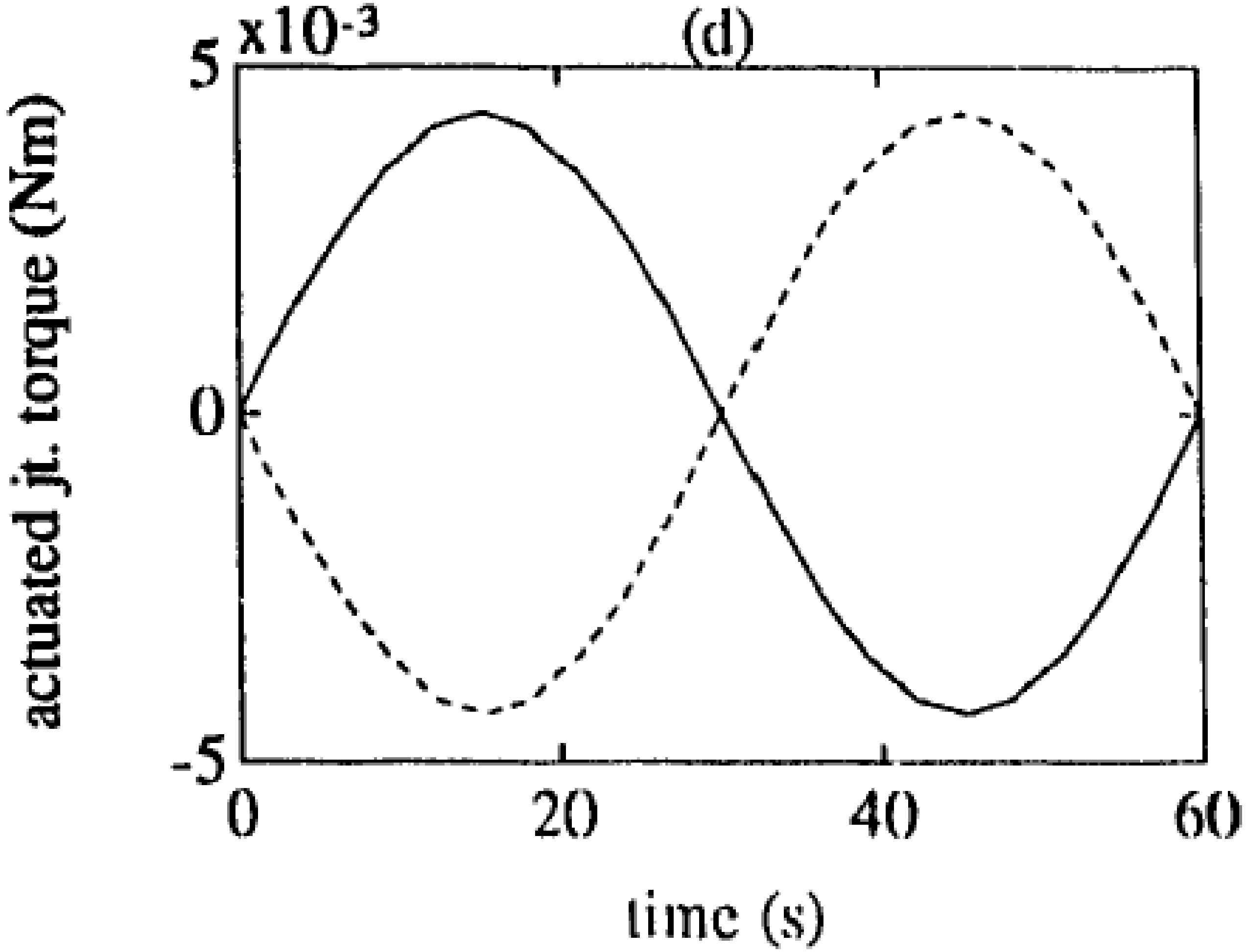
Joint Torques@**i** (-1&2; .. 3&4



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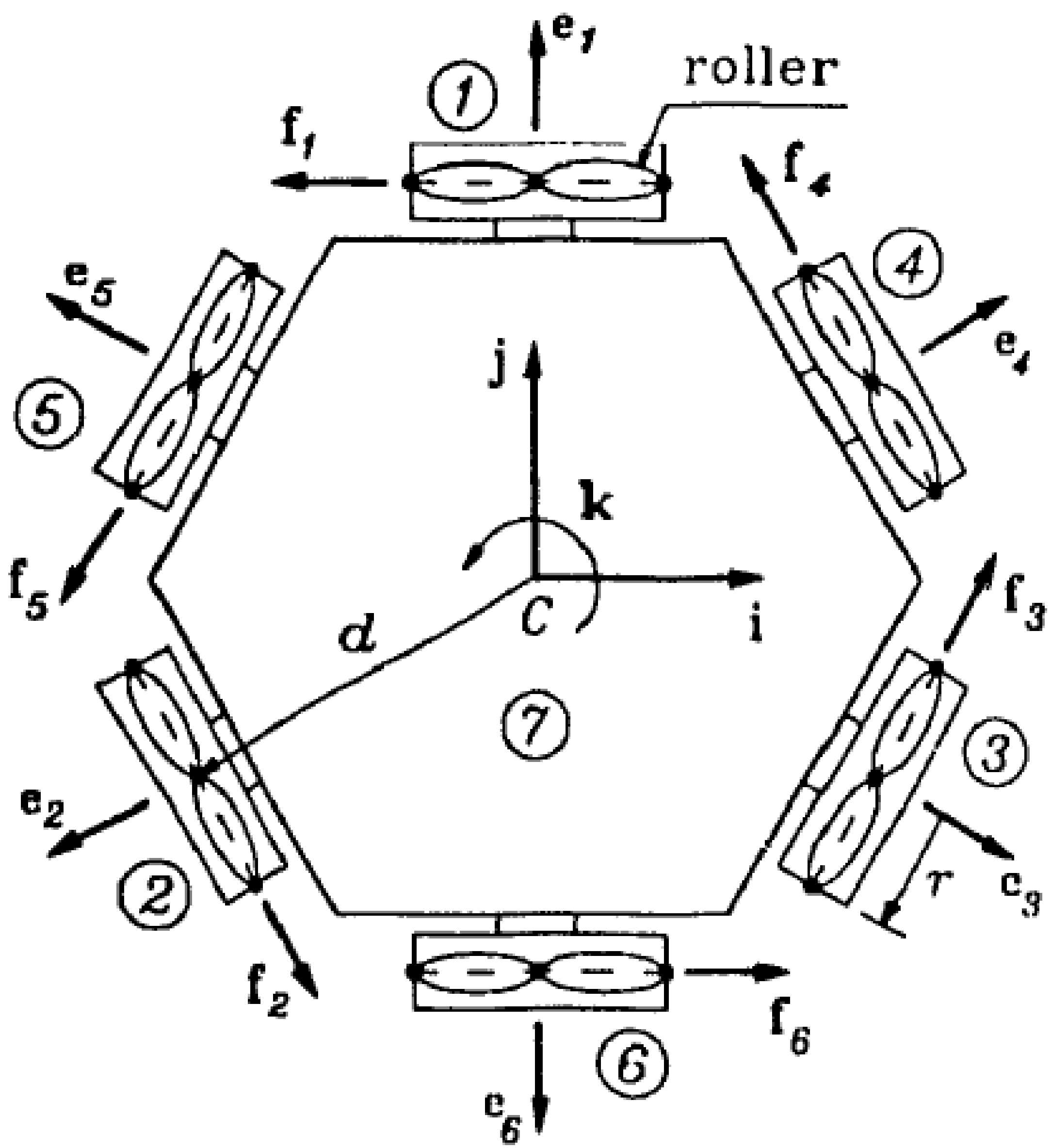
Three-DOF 6-Wheeled



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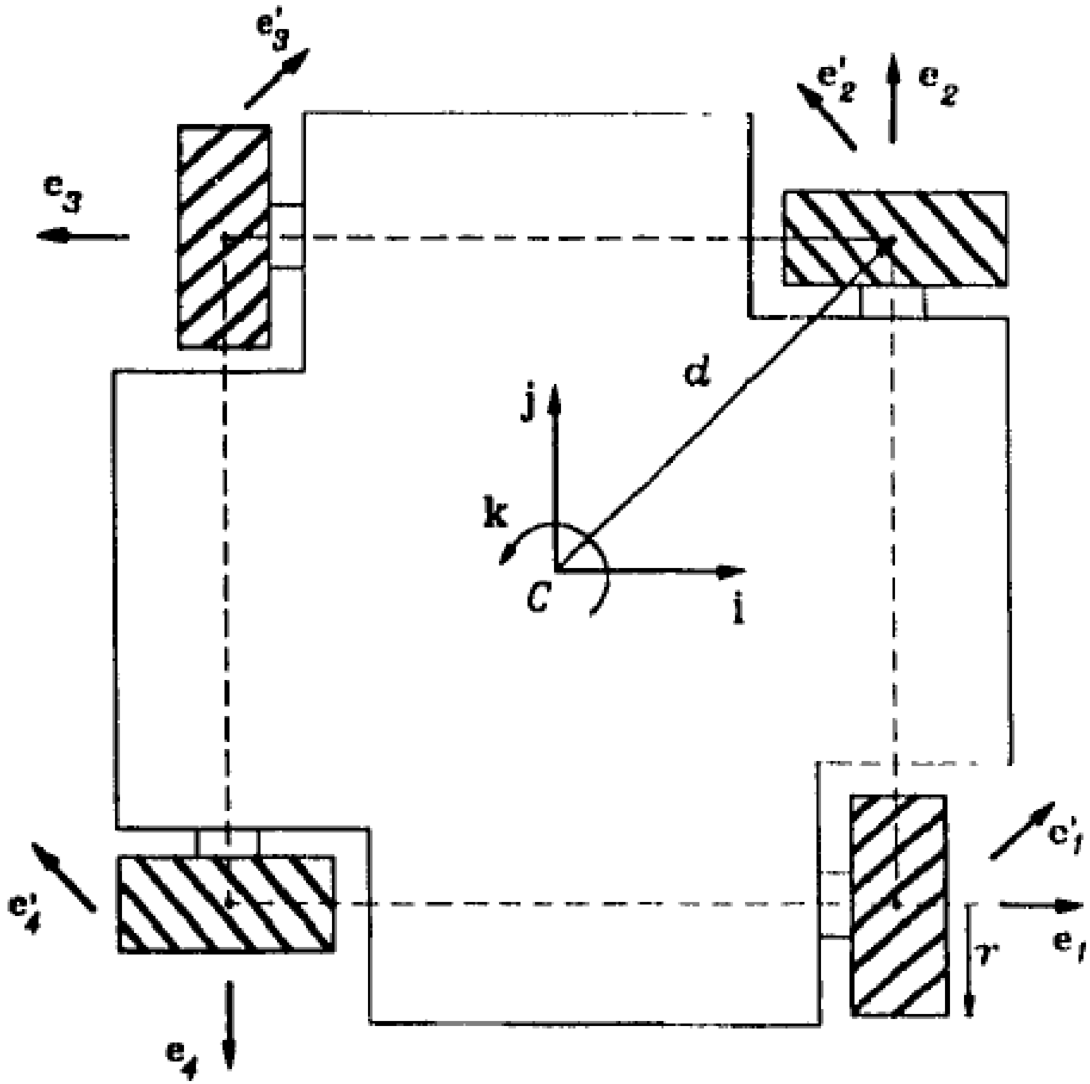
Isotropic 3-DOF 4-Wheeled



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Summary

- Mobile robots are nonholonomic systems
- Dynamic model for a 2-wheeled system
- Several mobile robots with omnidirectional wheels are shown
- Isotropic design was emphasized

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