JRL301
Robotics Technology

Minor I Questions

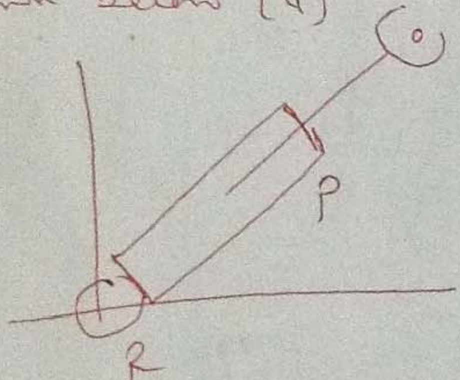
Sept. 7, 2016

1. Answer the following questions: (2×2 = 4)

   a. Express the Homogeneous Transformation matrix between two consecutive links of a serial arm in terms of four Denavit - Hartenberg (DH) parameters.

   b. What is duality of the Jacobian matrix in a serial robot?

2. Write forward kinematics relation for the Revolute (R) - Prismatic (P) jointed robot arm shown below [4]



Answers.

$$T_b T_\theta = \begin{bmatrix} c\theta & -s\theta & 0 & 0 \\ s\theta & c\theta & 0 & 0 \\ 0 & 0 & 1 & b \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_a T_\alpha = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & c\alpha & -s\alpha & 0 \\ 0 & s\alpha & c\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1.a. $T_b = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & b \\ 0 & 0 & 0 & 1 \end{bmatrix}$; $T_\theta = \begin{bmatrix} c\theta & -s\theta & 0 & 0 \\ s\theta & c\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (1)

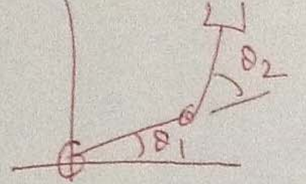
$T_a = \begin{bmatrix} 1_{3\times3} & \begin{matrix} a \\ 0 \\ 0 \end{matrix} \\ 0 \; 0 \; 0 & 1 \end{bmatrix}$; $T_\alpha = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha & -s\alpha & 0 \\ 0 & +s\alpha & c\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$T = T_b T_\theta T_a T_\alpha = \begin{bmatrix} c\theta & -s\theta c\alpha & s\theta s\alpha & a c\theta \\ s\theta & c\theta c\alpha & -c\theta s\alpha & a s\theta \\ 0 & s\alpha & c\alpha & b \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ (1)

(PTO)

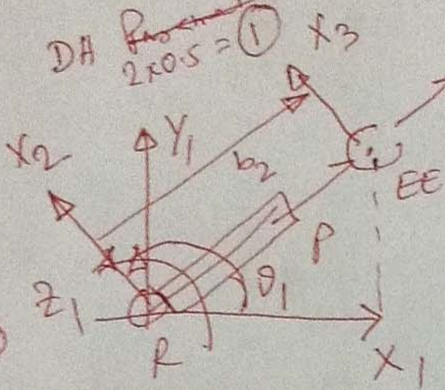
b) $\vec{t}_e = \underbrace{J}_{\text{Jacobian}} \dot{\vec{\theta}} \Rightarrow \vec{\tau} = J^T \vec{w}_e$

(1) (1)



DA ~~Parameters~~ Frame $2 \times 0.5 = $ (1)

2.



$2 \times 0.5 = $ (1)

i	b	θ	a	α
1	0	θ_1	0	$\pi/2$
2	b_2	0	0	0

$$\boxed{\begin{aligned} x_e &= b_2 s_1 \\ y_e &= -b_2 c_1 \end{aligned}}$$

$x_e = b_2 c(\theta_1 - 90°) \Rightarrow$ $\dot{x}_e = \dot{b}_2 c\theta_1 - b_2 s\theta_1 \dot{\theta}_1$

$y_e = b_2 s(\theta_1 - 90°) \Rightarrow$ $\dot{y}_e = \dot{b}_2 s\theta_1 + b_2 c\theta_1 \dot{\theta}_1$

Not asked

$$\begin{bmatrix} \dot{x}_e \\ \dot{y}_e \end{bmatrix} = \underbrace{\begin{bmatrix} -b_2 s\theta_1 & c\theta_1 \\ b_2 c\theta_1 & s\theta_1 \end{bmatrix}}_{J} \begin{bmatrix} \dot{\theta}_1 \\ \dot{b}_2 \end{bmatrix}$$

$2 \times 0.5 = $ (1)

$$T_1 = \left[\begin{array}{ccc|c} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \qquad T_2 = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & b_2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

(1) $T = T_1 T_2 = \left[\begin{array}{c} \cancel{b_2 s_1} \\ \cancel{-b_2 c_1} \\ \cancel{0} \end{array}\right] \left[\begin{array}{ccc|c} c_1 & 0 & s_1 & b_2 s_1 \\ s_1 & 0 & -c_1 & -b_2 c_1 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$