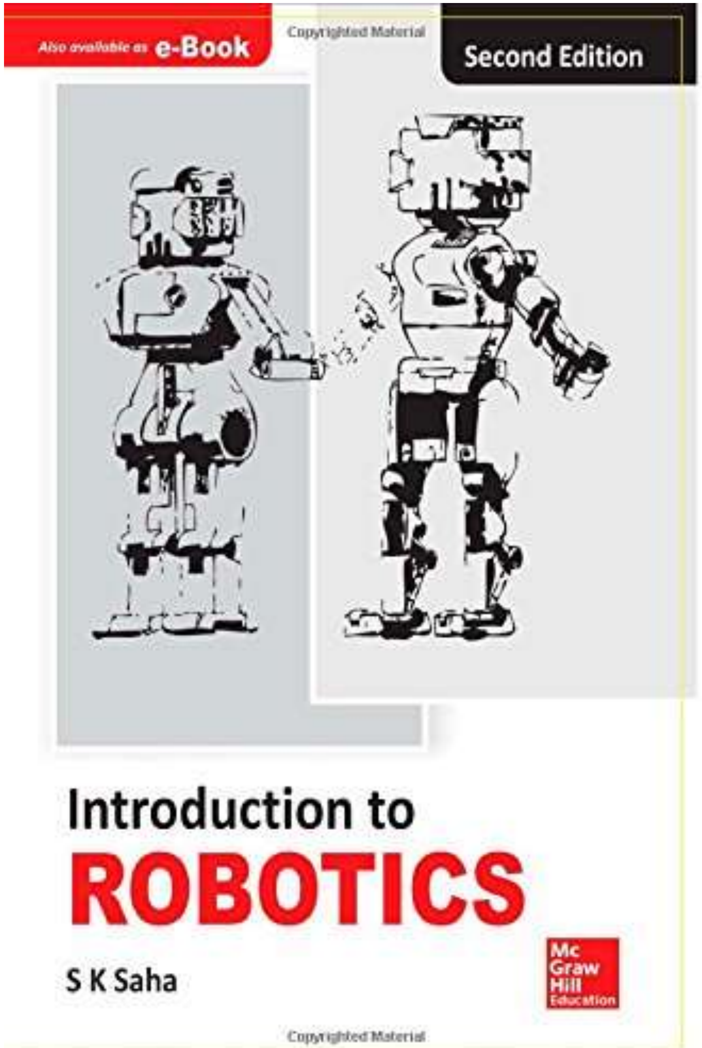
McGraw Hill Education



Lecture 2

Robot Kinematics (Ch. 5)

by

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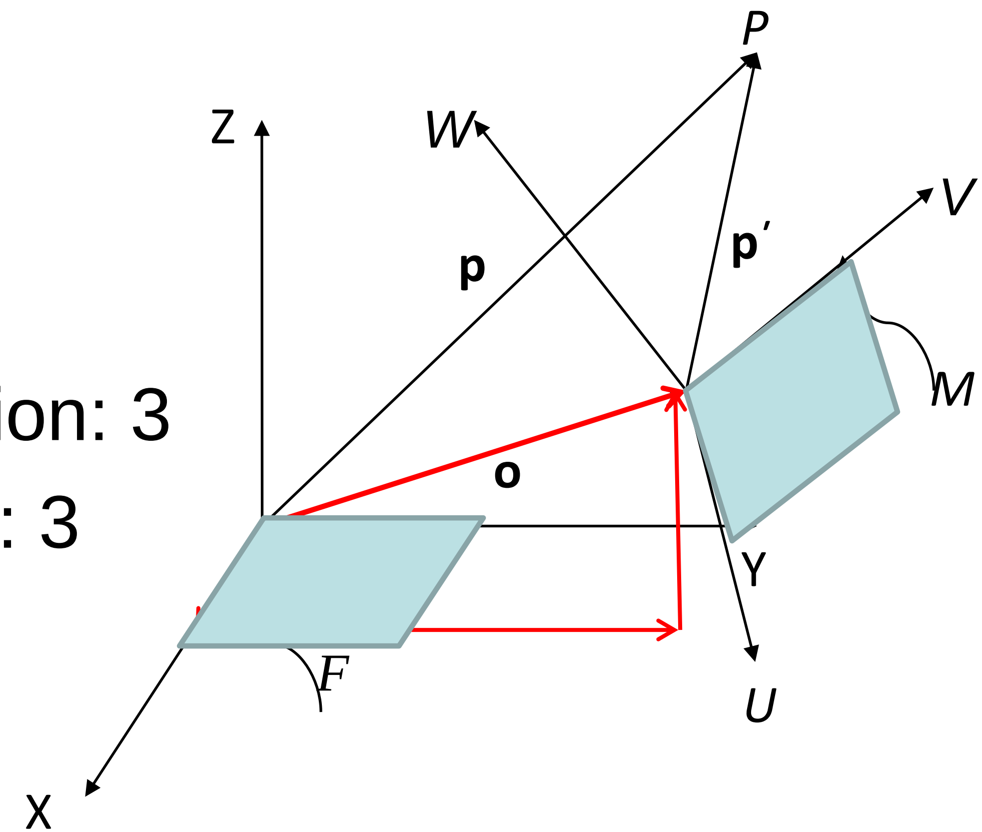
Aug. 08, 2017 (Tu)@JRL301 (Rob. Tech.)

5

Transformations

Spatial Motion

Pose ≡ Position + Rotation



Translation: 3

Rotation: 3

Total: 6

A moving body → Pose or Configuration

Position Description

$$[\mathbf{p}]_F \equiv \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} \quad \ldots (5.8)$$

$$\mathbf{p} = p_x\,\mathbf{x} + p_y\,\mathbf{y} + p_z\,\mathbf{z} \quad \ldots (5.9)$$

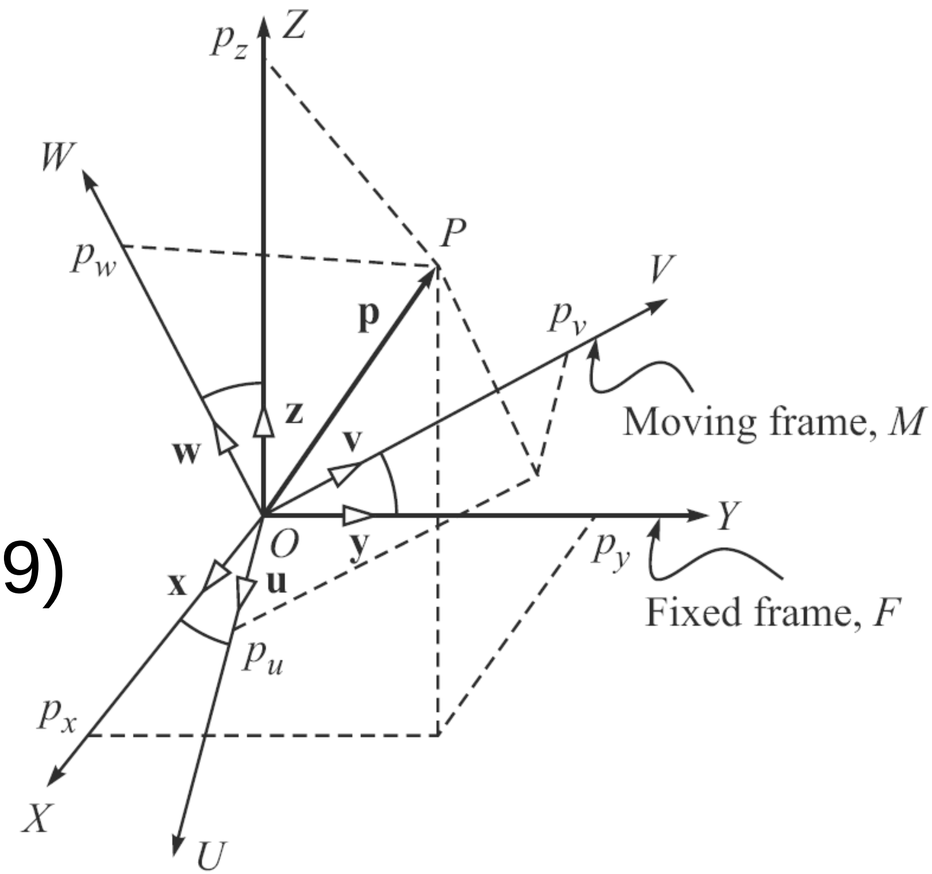


Fig. 5.12 Spatial description

$$[\mathbf{x}]_F \equiv \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \; [\mathbf{y}]_F \equiv \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \text{ and } [\mathbf{z}]_F \equiv \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \ldots (5.10)$$

Orientation Description

1. Direction cosine representation
2. Fixed-axes rotations
3. Euler angles representation
4. Single- and double-axes rotations
5. Euler parameters representation

I will illustrate 1 and 3 only

Direction Cosine Representation

Refer to Fig. 5.12

$$\mathbf{p} = p_u\mathbf{u} + p_v\mathbf{v} + p_w\mathbf{w} \quad \ldots (5.12)$$

$$\mathbf{u} = u_x\,\mathbf{x} + u_y\,\mathbf{y} + u_z\,\mathbf{z} \quad \ldots (5.11a)$$

$$\mathbf{v} = v_x\,\mathbf{x} + v_y\,\mathbf{y} + v_z\,\mathbf{z} \quad \ldots (5.11b)$$

$$\mathbf{w} = w_x\,\mathbf{x} + w_y\,\mathbf{y} + w_z\,\mathbf{z} \quad \ldots (5.11c)$$

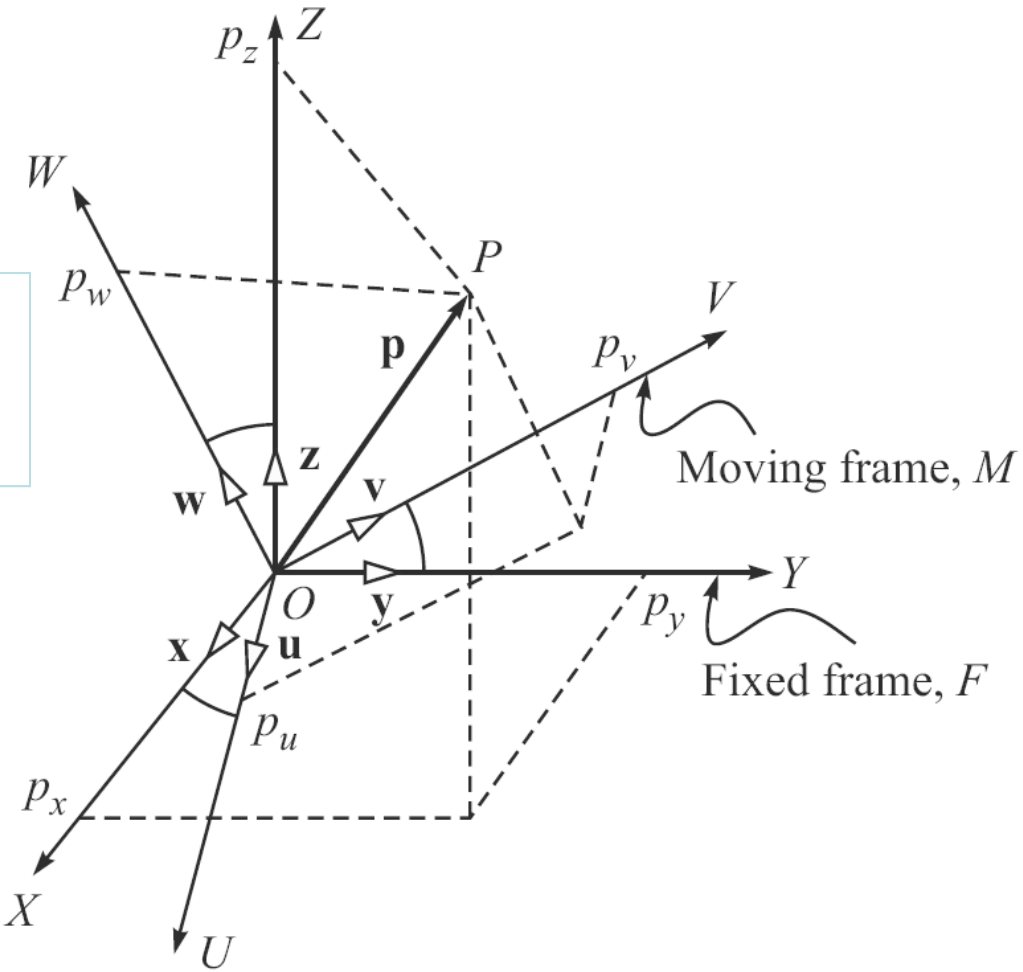


Fig. 5.12 Spatial description

Substitute eqs. (5.11a-c) into eq. (5.12)

$$\mathbf{p} = (p_u u_x + p_v v_x + p_w w_x)\mathbf{x} + (p_u u_y + p_v v_y + p_w w_y)\mathbf{y} + (p_u u_z + p_v v_z + p_w w_z)\mathbf{z} \qquad \ldots (5.13)$$

$$p_x = u_x p_u + v_x p_v + w_x p_w \qquad \ldots (5.14a)$$

$$p_y = u_y p_u + v_y p_v + w_y p_w \qquad \ldots (5.14b)$$

$$p_z = u_z p_u + v_z p_v + w_z p_w \qquad \ldots (5.14c)$$

$$[\mathbf{p}]_F = \mathbf{Q}\,[\mathbf{p}]_M \qquad \ldots (5.15)$$

$$[\mathbf{p}]_F = \mathbf{Q}\,[\mathbf{p}]_M \quad \ldots (5.15)$$

$$[\mathbf{p}]_F \equiv \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}, \quad [\mathbf{p}]_M \equiv \begin{bmatrix} p_u \\ p_v \\ p_w \end{bmatrix}, \quad \mathbf{Q} \equiv \begin{bmatrix} u_x & v_x & w_x \\ u_y & v_y & w_y \\ u_z & v_z & w_z \end{bmatrix} = \begin{bmatrix} \mathbf{u}^T\mathbf{x} & \mathbf{v}^T\mathbf{x} & \mathbf{w}^T\mathbf{x} \\ \mathbf{u}^T\mathbf{y} & \mathbf{v}^T\mathbf{y} & \mathbf{w}^T\mathbf{y} \\ \mathbf{u}^T\mathbf{z} & \mathbf{v}^T\mathbf{z} & \mathbf{w}^T\mathbf{z} \end{bmatrix} \quad \ldots (5.16)$$

Orientation description 1

$\mathbf{u}^T\mathbf{u} = \mathbf{v}^T\mathbf{v} = \mathbf{w}^T\mathbf{w} = 1$, and

$$\mathbf{u}^T\mathbf{v}(\equiv\mathbf{v}^T\mathbf{u}) = \mathbf{u}^T\mathbf{w}(\equiv\mathbf{w}^T\mathbf{u}) = \mathbf{v}^T\mathbf{w}(\equiv\mathbf{w}^T\mathbf{v}) = 0 \quad \ldots (5.17)$$

Q is called Orthogonal

Due to orthogonality

$$\mathbf{u} \times \mathbf{v} = \mathbf{w}, \quad \mathbf{v} \times \mathbf{w} = \mathbf{u}, \text{ and } \quad \mathbf{w} \times \mathbf{u} = \mathbf{v} \quad \ldots (5.18)$$

$$\mathbf{Q}^T\mathbf{Q} = \mathbf{Q}\mathbf{Q}^T = \mathbf{1}\,; \;\; \det(\mathbf{Q}) = 1; \;\; \mathbf{Q}^{-1} = \mathbf{Q}^T \ \ldots (5.19)$$

Example 5.6 Rotations [Elementary] (Fig. 5.13a)

Interpretation 1

$$[\mathbf{u}]_F \equiv \begin{bmatrix} C\alpha \\ S\alpha \\ 0 \end{bmatrix}, \quad [\mathbf{v}]_F \equiv \begin{bmatrix} -S\alpha \\ C\alpha \\ 0 \end{bmatrix}, \quad [\mathbf{w}]_F \equiv \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \ldots (5.20)$$

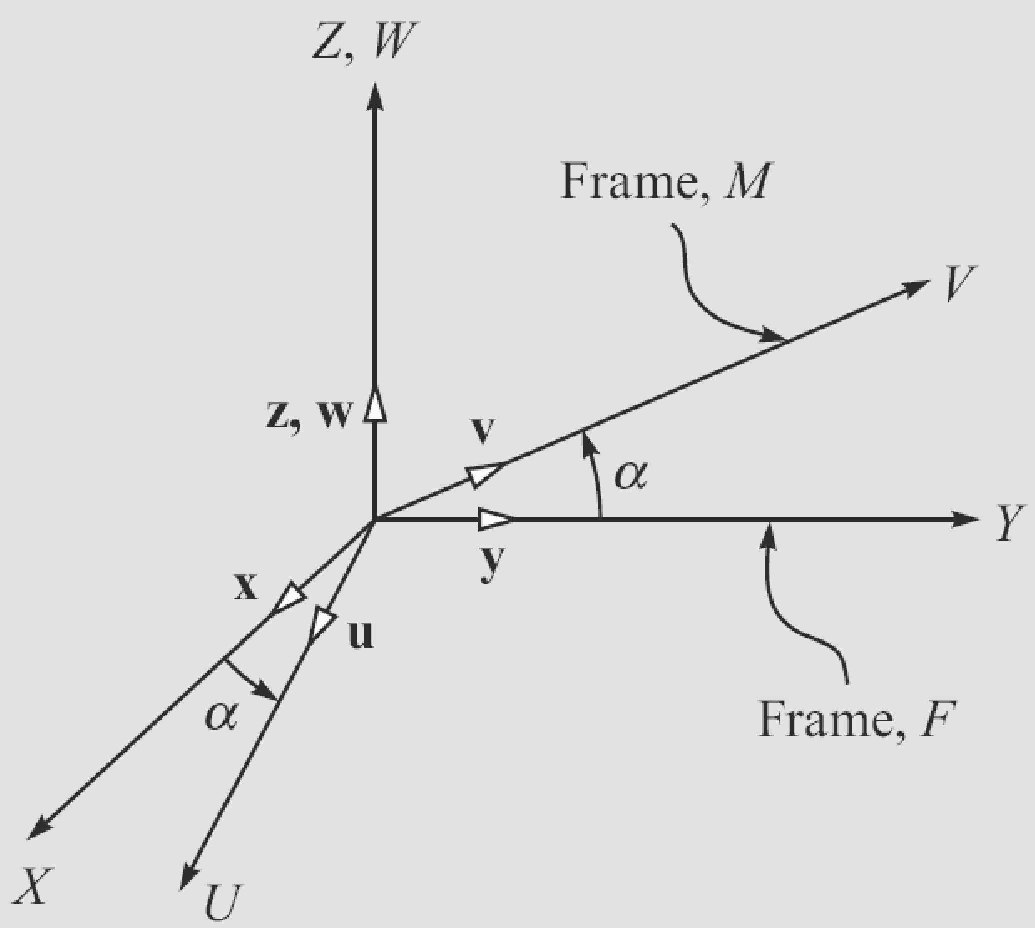


Fig. 5.13 (a) Rotation about *Z*-axis

$$\mathbf{Q}_Z \equiv \begin{bmatrix} C\alpha & -S\alpha & 0 \\ S\alpha & C\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \ldots (5.21)$$

$$\mathbf{Q}_Y \equiv \begin{bmatrix} C\beta & 0 & S\beta \\ 0 & 1 & 0 \\ -S\beta & 0 & C\beta \end{bmatrix}; \quad \mathbf{Q}_X \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\gamma & -S\gamma \\ 0 & S\gamma & C\gamma \end{bmatrix}$$

$$\ldots (5.22)$$

Non-commutative Property: An Illustration

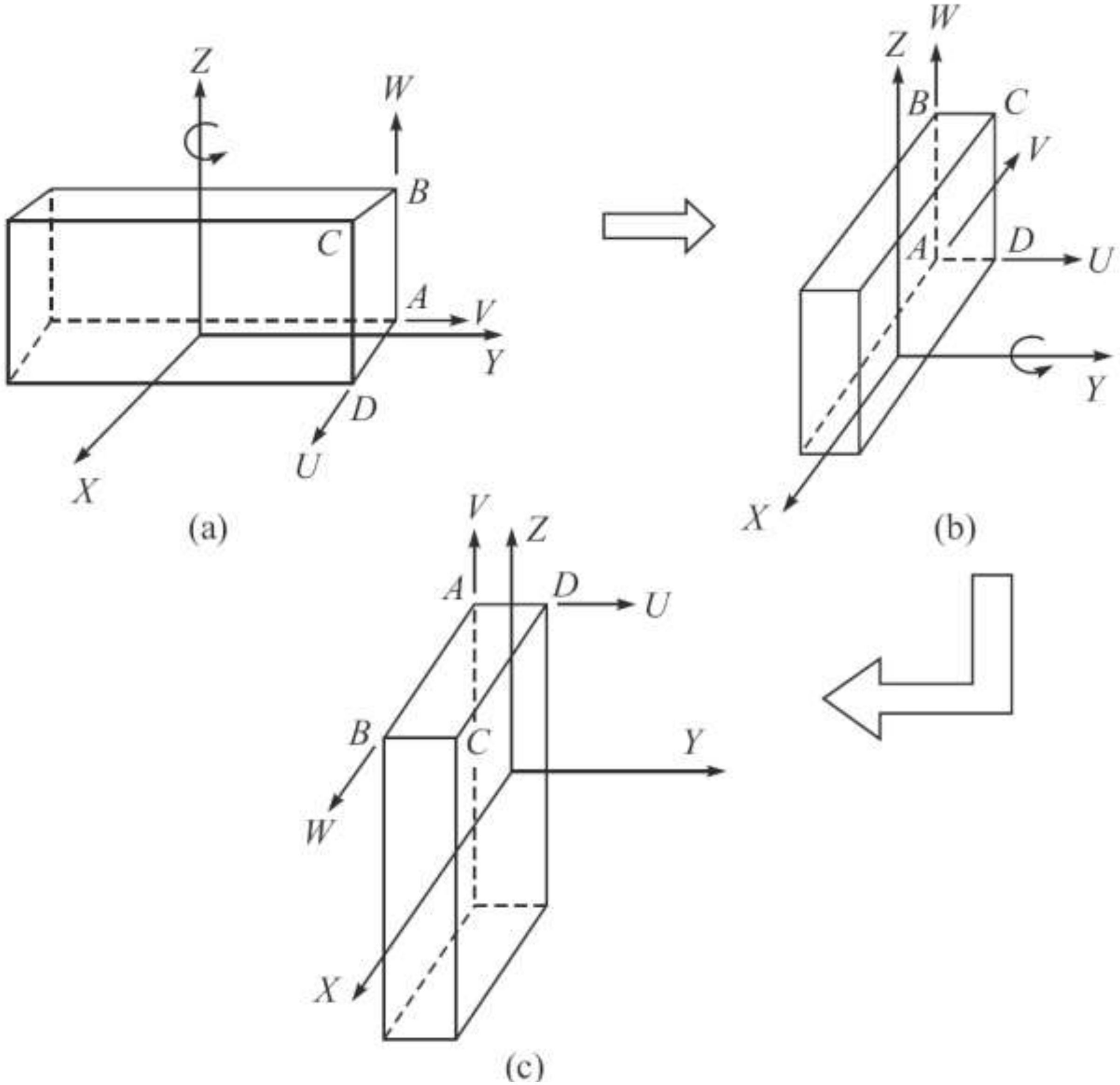


Fig. 5.20 Successive rotation of a box about Z and Y-axes

Non-commutative Property (contd.)

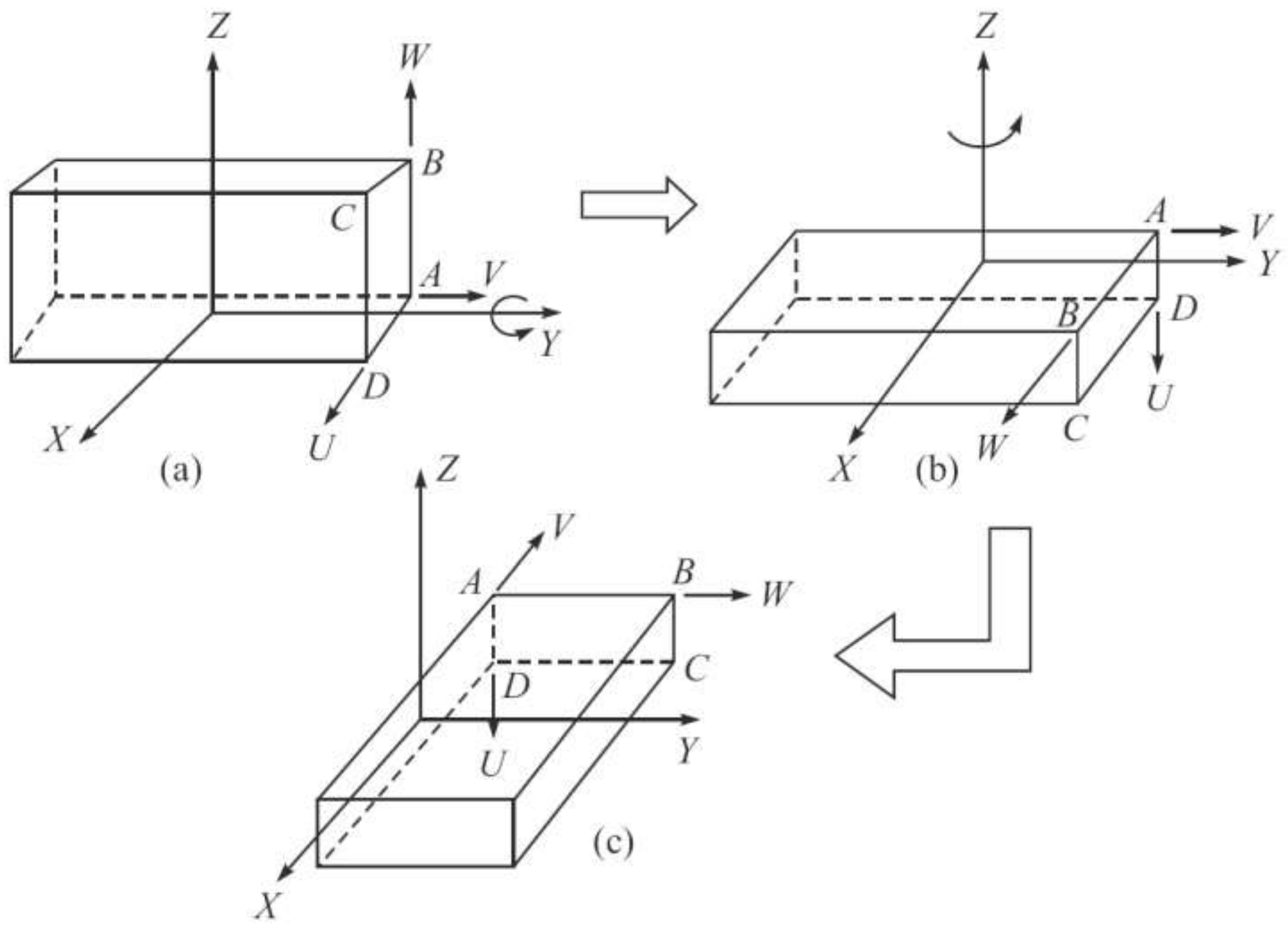
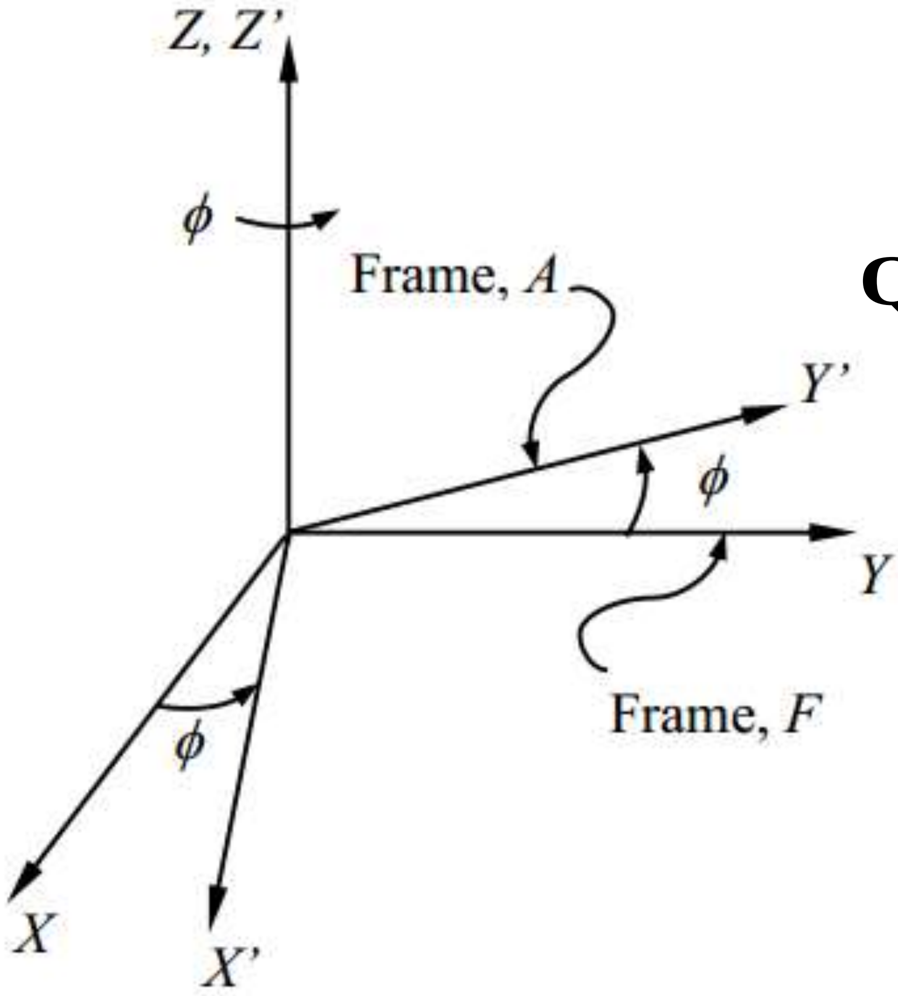


Fig. 5.21 Successive rotation of a box about Y and Z-axes

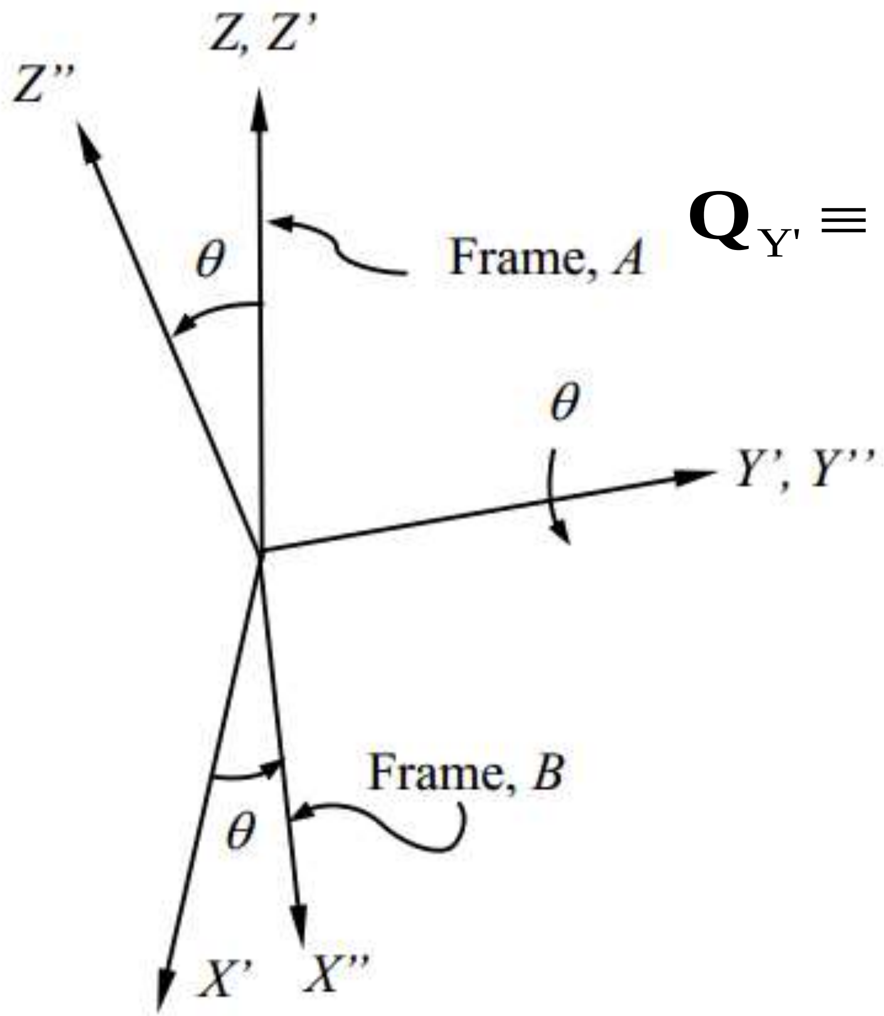
Euler Angles Representation (ZYZ)



$$\mathbf{Q}_Z \equiv \begin{bmatrix} C\phi & -S\phi & 0 \\ S\phi & C\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \ldots(5.34a)$$

Fig. 5.16 (a) Rotation about *Z*-axis

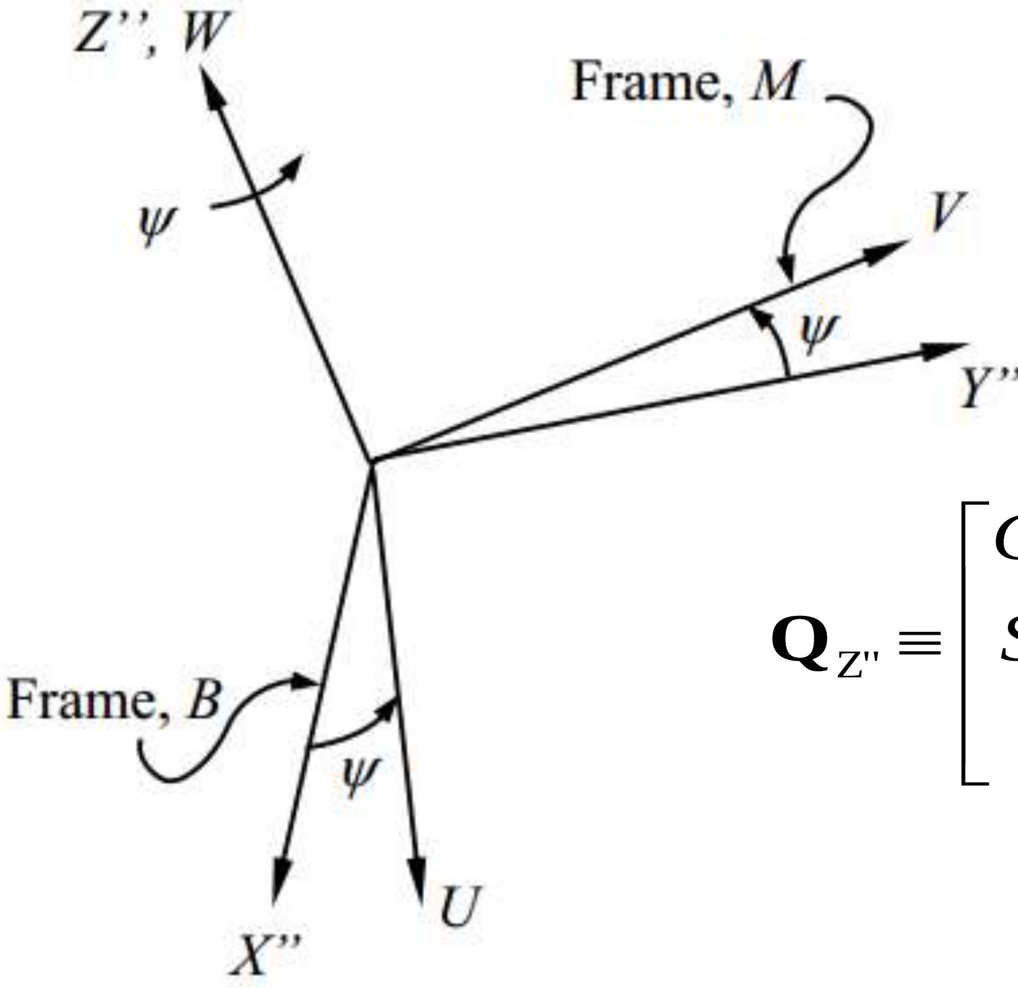
Euler Angles Representation …



$$\mathbf{Q}_{Y'} \equiv \begin{bmatrix} C\theta & 0 & S\theta \\ 0 & 1 & 0 \\ -S\theta & 0 & C\theta \end{bmatrix} \quad \ldots(5.34b)$$

Fig. 5.16 (b) Rotation about current *Y*-axis, i.e., *Y'*-axis

Euler Angles Representation …



$$\mathbf{Q}_{Z''} \equiv \begin{bmatrix} C\psi & -S\psi & 0 \\ S\psi & C\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \ldots(5.34c)$$

Fig. 5.16 (c) Rotation about current *Z*-axis, i.e., *Z''*-axis

$$\mathbf{Q} = \mathbf{Q}_Z \mathbf{Q}_{Y'} \mathbf{Q}_{Z''} \quad \ldots (5.34d)$$

$$\mathbf{Q} \equiv \begin{bmatrix} C\phi C\theta C\psi - S\phi S\psi & -C\phi C\theta S\psi - S\phi C\psi & C\phi S\theta \\ S\phi C\theta C\psi + C\phi S\psi & -S\phi C\theta S\psi + C\phi C\psi & S\phi S\theta \\ -S\theta C\psi & S\theta S\psi & C\theta \end{bmatrix}$$

. . . (5.34e)

For extraction purpose, say, input is given by

$$\mathbf{Q} \equiv \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{bmatrix} \quad \ldots (5.35a)$$

$$\phi = a\tan 2\left(\frac{q_{23}}{S\theta}, \frac{q_{13}}{S\theta}\right) \quad \ldots (5.35b)$$

Cannot find ϕ when $\theta = 0$ or π

Example 5.11 Extract ZYZ Euler Angles (Fig. 5.18)

$$\mathbf{Q} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix} \quad \ldots (5.36)$$

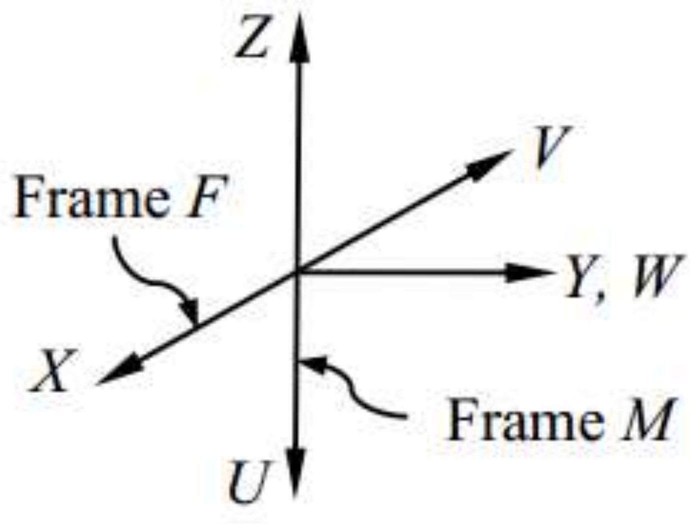


Fig. 5.18 Relation between frames *M* and *F*

- The (3,3) element: $C\theta = 0 \Rightarrow \theta = 90°$
- The (1,3) and (2,3) elements: $S\theta C\phi = 0$; $S\theta S\phi = 1 \Rightarrow S\phi = 1$ (since $S\theta = 1$) $\Rightarrow \phi = 90°$
- The (3,1) element: $-S\theta C\psi = -1 \Rightarrow \psi = 0°$

Summary

- Direction cosine representation
- Euler angles and associated transformation

THANK YOU

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