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Fundamentals

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Announcement

- Outlines of Lecture 02 will be uploaded to

  http://sksaha.com/courses

Review of Lecture 2

- **Robot classifications based on**
  - **Application. Coordinate system, Control, Programming, etc.**

Outline

- **Mathematical Fundamentals**
  - **Vectors and matrices**
- **Manipulator**
  - **Links**
  - **Joints**
- **Degrees of freedom**
  - **Definition**
  - **Formula**

Vectors

- Array of n-numbers written column-wise (not row-wise)

$$\mathbf{a} \equiv \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}$$

- If a row-vector is needed, use transpose

$$\begin{bmatrix} a_1, & \cdots, & a_n \end{bmatrix} \equiv \mathbf{a}^T$$

Length and Direction

- Length or magnitude or norm of a vector

$$a = \sqrt{\mathbf{a}^T \mathbf{a}} = \sqrt{a_1^2 + \cdots + a_n^2}$$

- For a Cartesian position vector, length

$$a = \sqrt{\mathbf{a}^T \mathbf{a}} = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

- Direction (e.g., *Angle* with XY plane)

$$Angle = a\tan 2(a_3, \sqrt{a_1^2 + a_2^2})$$

Unit Vector

- A vector divided by its lengths, $\bar{\mathbf{a}} = \dfrac{\mathbf{a}}{a}$

- Examples of unit vectors

$$\mathbf{i} \equiv \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{j} \equiv \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad \mathbf{k} \equiv \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

- Any vector can be represented as

$$\mathbf{a} = a_1\,\mathbf{i} + \mathrm{a}_2\,\mathbf{j} + a_3\,\mathbf{k}$$

Scalar and Dot Products

- Meaning projections

$$\mathbf{a} \cdot \mathbf{b} \equiv \mathbf{a}^T \mathbf{b} = ab \cos \theta$$

$$(\mathbf{a}^T \mathbf{b} = \mathbf{b}^T \mathbf{a})$$

- Alternate way of calculations

$$\mathbf{a}^T \mathbf{b} = a_1 b_1 + \cdots + a_n b_n$$

Vector- or Cross-product

- Definition

$$\mathbf{c} = \mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$\mathbf{c} = (a_2 b_3 - a_3 b_2)\mathbf{i} + (a_3 b_1 - a_1 b_3)\mathbf{j} + (a_1 b_2 - a_2 b_1)\mathbf{k}$$

- Magnitude

$$c = |\mathbf{a} \times \mathbf{b}| = ab \sin\theta$$

Properties of Cross-product

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a}^T \mathbf{c})\mathbf{b} - (\mathbf{a}^T \mathbf{b})\mathbf{c}$$

$$\mathbf{a}^T (\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} \mathbf{a} & \mathbf{b} & \mathbf{c} \end{vmatrix}$$

$$(\mathbf{a} \times \mathbf{1})\mathbf{b} = \mathbf{a} \times \mathbf{b}$$

$$(\mathbf{a} \times \mathbf{1}) \equiv \mathbf{A} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

- **A** is called cross-product matrix

Differentiation of a Vector

$$\frac{d\mathbf{a}}{dt} \equiv \dot{\mathbf{a}} \equiv [\dot{a}_1 \quad \cdots \quad \dot{a}_n]^T$$

- Chain rules of differentiation

$$\frac{d}{dt}(\mathbf{a}^T\mathbf{b}) = \dot{\mathbf{a}}^T\mathbf{b} + \mathbf{a}^T\dot{\mathbf{b}}$$

$$\frac{d}{dt}(\mathbf{a}\times\mathbf{b}) = \dot{\mathbf{a}}\times\mathbf{b} + \mathbf{a}\times\dot{\mathbf{b}}$$

Linear Independence

- For a set of n independent vectors

$$\sum_{i=1}^{n} \alpha_i \mathbf{a}_i = \mathbf{0} \quad \Rightarrow \quad \alpha_i = 0 \quad \text{for all } i$$

Matrices

- For an m x n matrix

$$\mathbf{A} \equiv [\mathbf{a}_1 \quad \cdots \quad \mathbf{a}_n]$$

$$\mathbf{A} \equiv \begin{bmatrix} \mathbf{a}_1^T \\ \vdots \\ \mathbf{a}_m^T \end{bmatrix}$$

Determinant

- For an n x n (square) matrix

$$\det(\mathbf{A}) = |\mathbf{A}| = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = \sum (-1)^{i+j} a_{ij} \det(\mathbf{A}_{ij})$$

- Example of a 3 x 3 (square) matrix

$$\det(\mathbf{A}) = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

Inverse

$$\mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})} Adj(\mathbf{A})$$

$$Adj(\mathbf{A}) = [(-1)^{i+j} \det(\mathbf{A}_{ij})]^T$$

- Generally the above is not used. Solution of linear equations are used
- Use Gaussian Elimination (GE) to solve

Joints or Kinematic Pairs

- Lower Pair
  - Surface contact: Hinge joint of a door
- Higher pair
  - Line or point contact: Roller or ball rolling
- Several Lower Pair Joints
  - Slides 1-8 of Chapter 5

Manipulator

- It has a series of links connected by joints → Kinematic Chain (KC)
- Simple: When each and every link is coupled to at most two other links
  - Open: If it contains only two links (end ones) that are connected to only one link → Manipulator
  - Closed: If each and every link coupled to two other links → Mechanism

Degrees of Freedom (DOF)

- No. of independent (or minimum) coordinates required to fully describe its pose or configuration
  - A rigid body in 3D space has 6 DOF
- Grubler formula (1917) for planar mechanisms
- Kutzbach formula (1929) for spatial mechanisms

Grubler-Kutzbach Criterion

$$n = s\,(r - 1) - c, \qquad c \equiv \sum_{i=1}^{p} c_{\mathrm{i}} \qquad \ldots (5.1)$$

s : dim. of working space

(Planar, s = 3; Spatial, s = 6);

r : no. of rigid bodies or links in the system;

p : no. of kinematic pairs or joints in the system;

c_i : no. of constraints imposed by each joint;

c : total no. of constraints imposed by p joints;

n_i : relative degree of freedom of each joint;

n : DOF of the whole system.

Basically, no. of parameters used to define free links – no. of constraints (independent) by joints

Four-bar Mechanism,

$$n = 3\,(4 - 4 - 1) + (1 + 1 + 1 + 1) = 1 \qquad \ldots (5.4)$$

Six-DOF Manipulator

$$n = 6\,(7 - 6 - 1) + 6 \times 1 = 6 \qquad \ldots (5.5)$$

Five-bar Mechanism

$$n = 3\,(5 - 5 - 1) + 5 \times 1 = 2 \qquad \ldots (5.6)$$

Double Parallelogram

$$n = 3\,(5 - 6 - 1) + 6 \times 1 = 0 \qquad \ldots (5.7)$$

Summary

- Vectors and matrices were defined
- Definitions of mechanisms, DOF, etc, were explained.

Thank You

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