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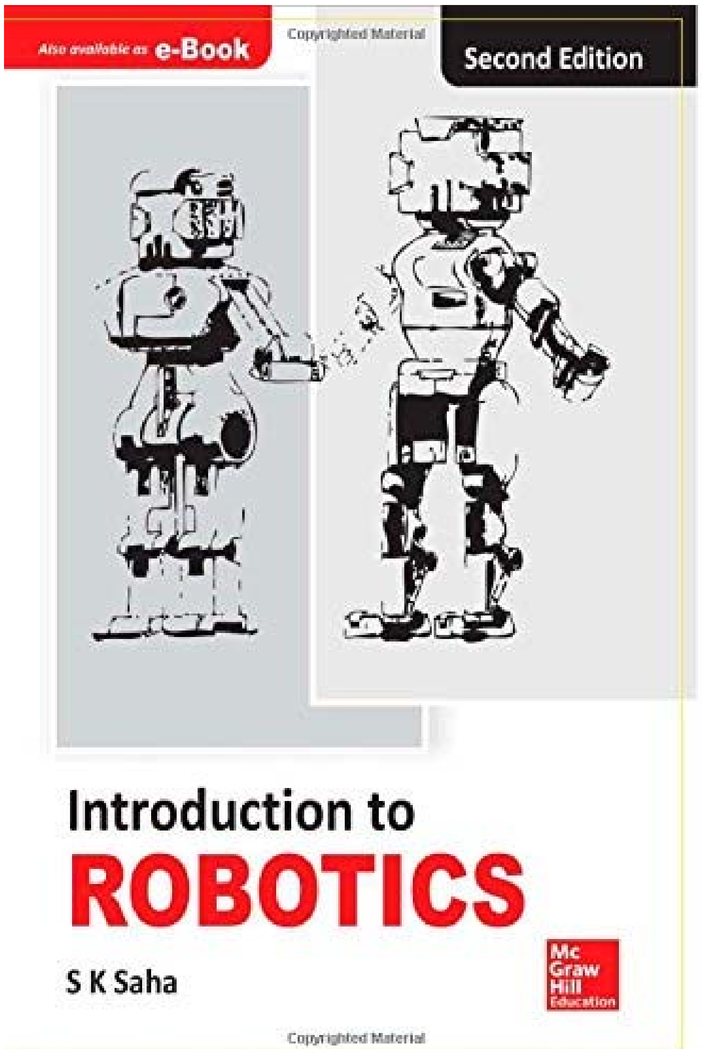
Education



Lecture 3

Robot Kinematics (Ch. 5)

by

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Aug. 22, 2017 (Tu)@JRL301 (Rob. Tech.)

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Recap

- Orientation representations
  - Non-commutative
- Direction cosines: Has disadv. of 9 param.
- Fixed-axes (RPY) rotations (12 sets)

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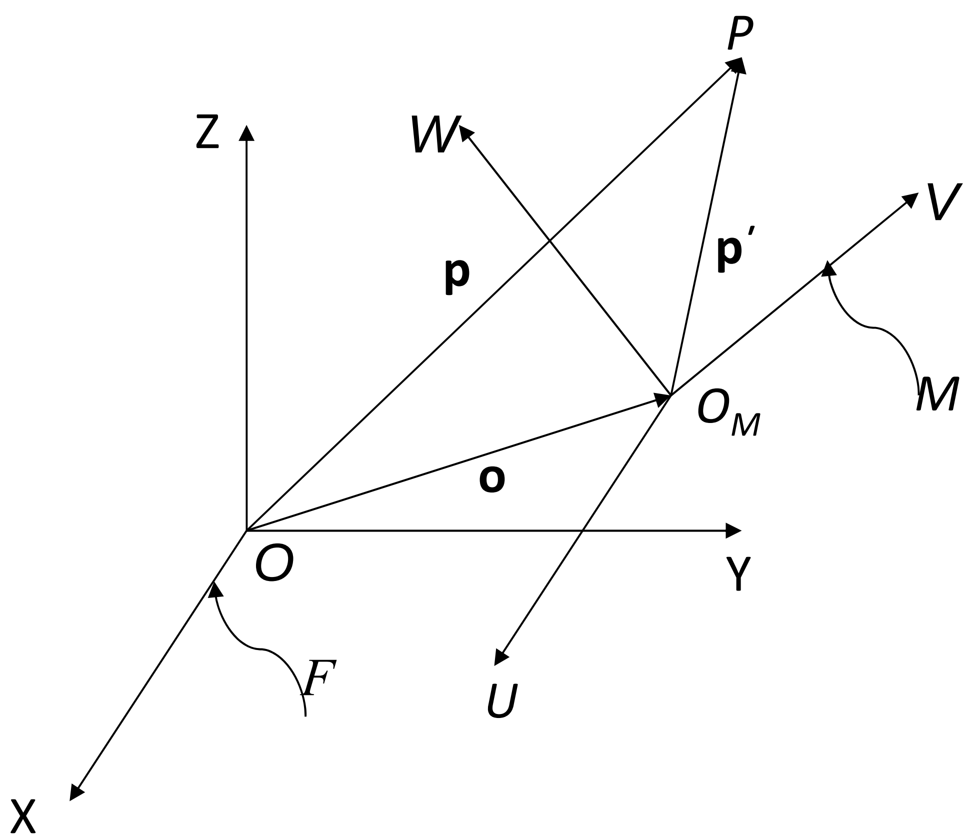
Homogeneous Transformation



Task: Point *P* is known in moving frame *M*. Find *P* in fixed frame *F*.

Fig. 5.23 Two coordinate frames

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$$\mathbf{p} = \mathbf{o} + \mathbf{p}' \qquad \ldots (5.45)$$

$$[\mathbf{p}]_F = [\mathbf{o}]_F + \mathbf{Q}[\mathbf{p}']_M \qquad \ldots (5.46)$$

$$\begin{bmatrix} [\mathbf{p}]_F \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{Q} & [\mathbf{o}]_F \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} [\mathbf{p}']_M \\ 1 \end{bmatrix} \qquad \ldots (5.47)$$

$$[\overline{\mathbf{p}}]_F = \mathbf{T}[\overline{\mathbf{p}'}]_M \qquad \ldots (5.48)$$

Homogenous Transformation

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T: Homogenous transformation matrix (4 × 4)

$$\mathbf{T}^T\mathbf{T} \neq \mathbf{1} \quad \text{or} \quad \mathbf{T}^{-1} \neq \mathbf{T}^T \qquad \ldots (5.49)$$

$$\mathbf{T}^{-1} = \begin{bmatrix} \mathbf{Q}^T & -\mathbf{Q}^T[\mathbf{o}]_F \\ \mathbf{0}^T & 1 \end{bmatrix} \qquad \ldots (5.50)$$

Example 5.10 Pure Translation

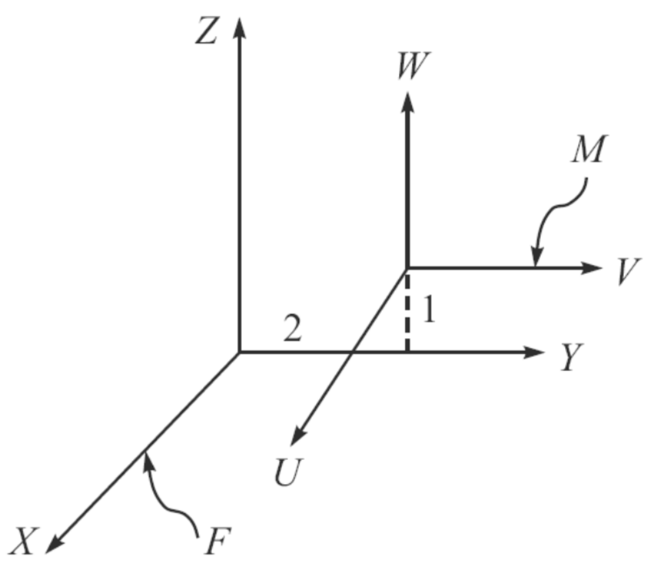

Fig. 5.24 (a)

$$\mathbf{T} \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \ldots (5.51)$$

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Example 5.11 Pure Rotation

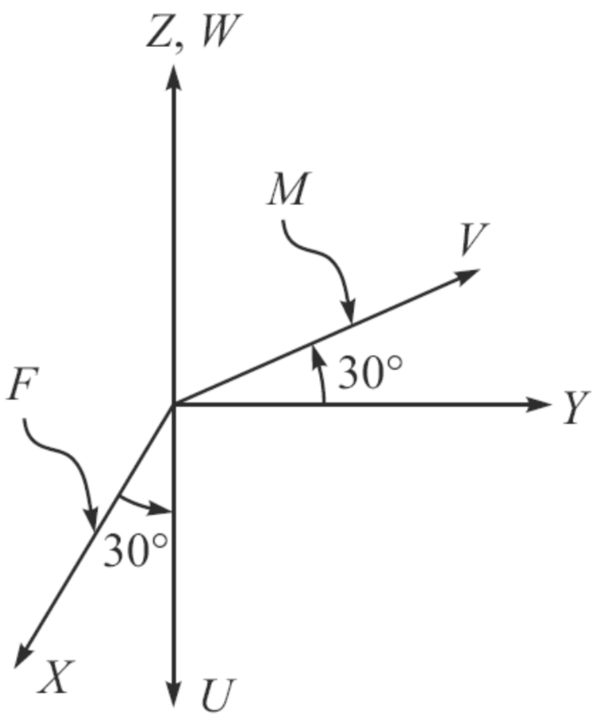

Fig. 5.24 (b)

$$\mathbf{T} \equiv \begin{bmatrix} C30^o & -S30^o & 0 & 0 \\ S30^o & C30^o & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{\frac{3}{2}} & -\frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \sqrt{\frac{3}{2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots (5.52)$$

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Non-commutative Property

Like rotation matrices homogeneous transformation matrices are non-commutative, i. e.,

$$\mathbf{T}_A\mathbf{T}_B \neq \mathbf{T}_B\mathbf{T}_A$$

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Denavit and Hartenberg (DH) Parameters

- Serial chain

- Two links connected by revolute or prismatic joint

- Four parameters
  - Joint offset (b)
  - Joint angle (θ)
  - Link length (a)
  - Twist angle (α)

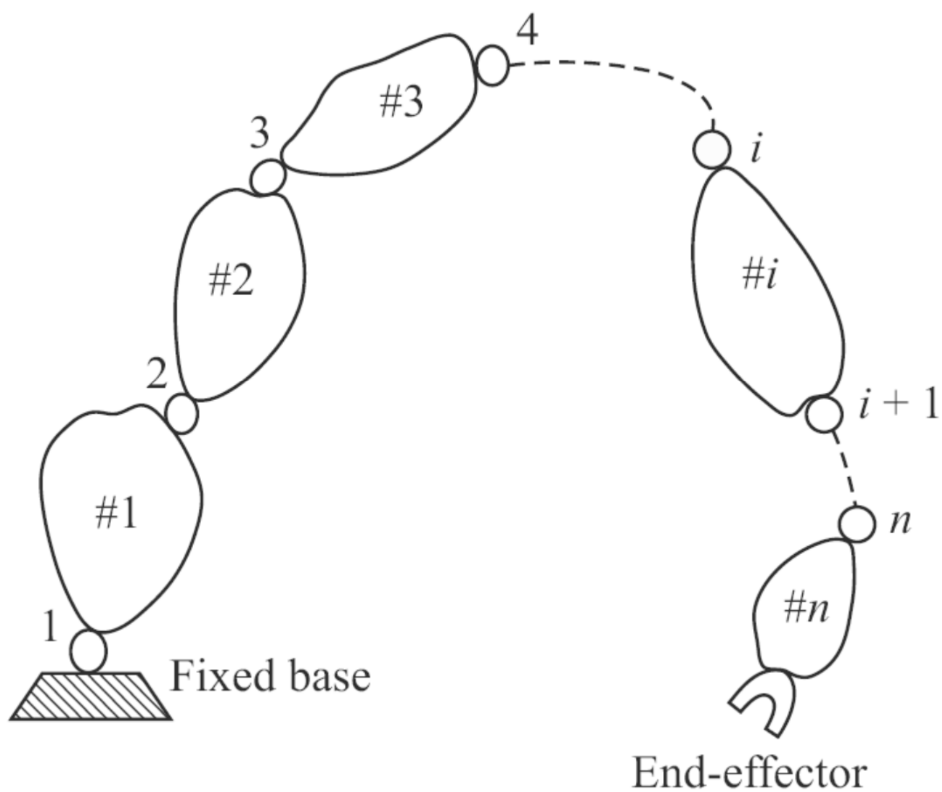

Fig. 5.27 Serial manipulator

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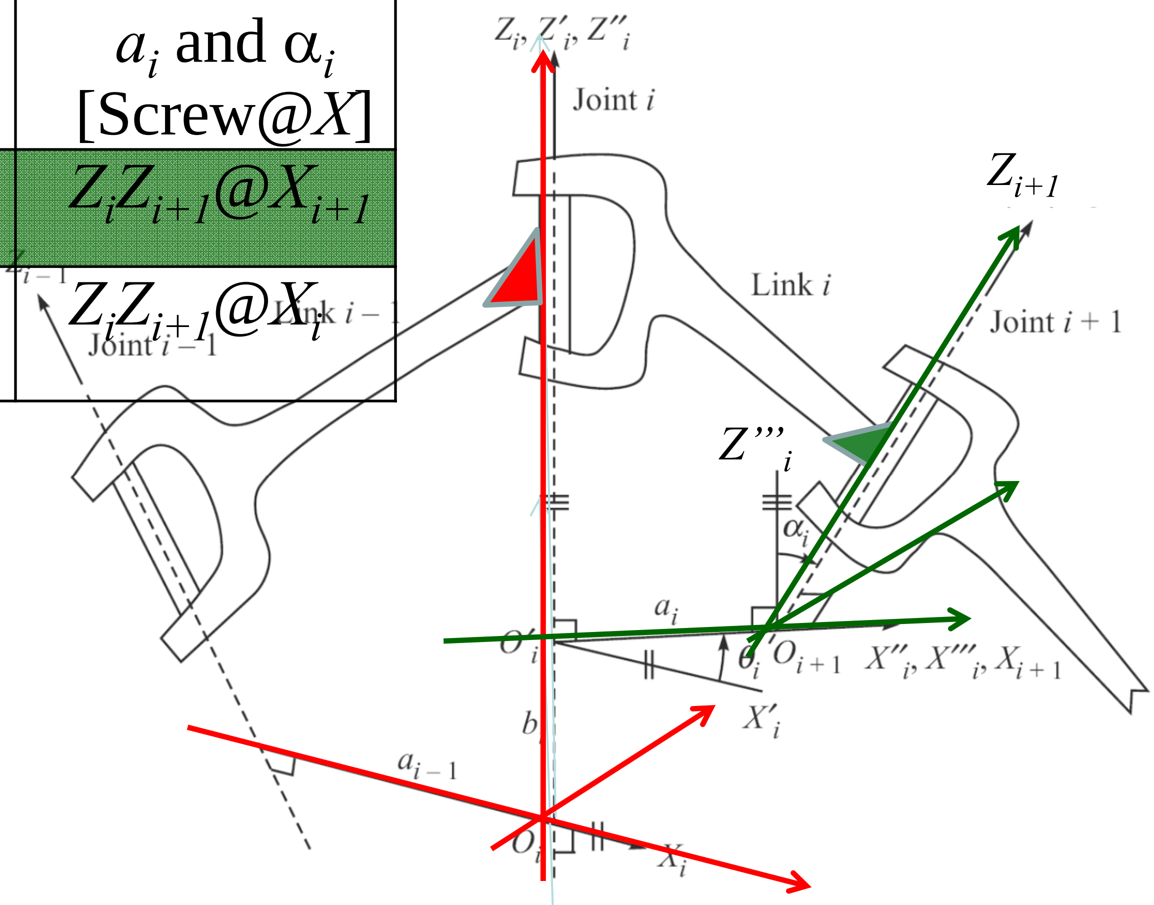
- <u>Joint axis *i*</u>: Link *i-1* + link *i*
- <u>Link *i*</u>: Fixed to **frame *i+1*** (Saha) / **frame *i*** (Craig)

DH	Variables b_i and θ_i [Screw@Z]	Constants a_i and α_i [Screw@X]
Saha	$X_iX_{i+1}@Z_i$	$Z_iZ_{i+1}@X_{i+1}$
Craig	$X_{i-1}X_i@Z_i$	$Z_iZ_{i+1}@X_i$



(a) The *i*th joint is revolute

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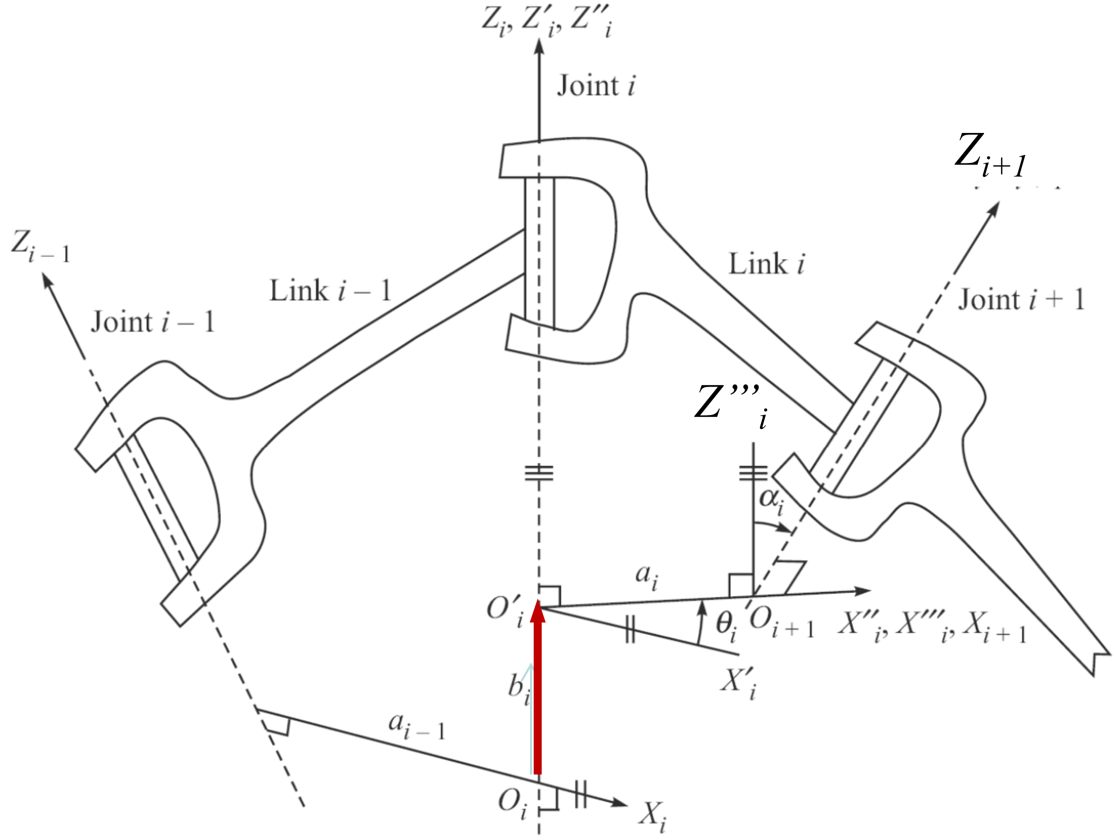
- b_i (Joint offset): Length of the intersections of the common normals on the joint axis Z_i, i.e., O_i and O'_i. It is the relative position of links i – 1 and i. This is measured as the distance between X_i and X_{i+1} along



(a) The ith joint is revolute

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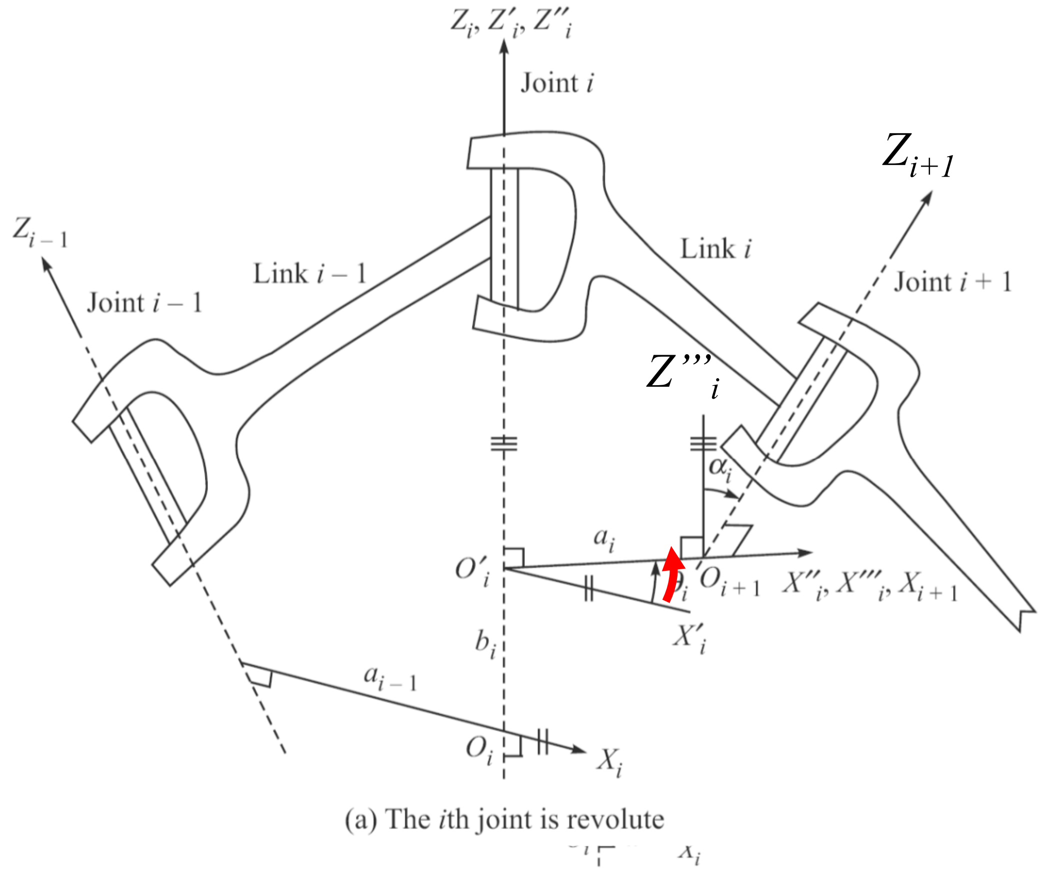
- θ_i (Joint angle): Angle between the orthogonal projections of the common normals, X_i and X_{i+1}, to a plane normal to the joint axes Z_i. Rotation is positive when it is made counter clockwise. It is the relative angle between links $i - 1$ and i. This is measured as the angle between X_i and X_{i+1} about Z_i.



(a) The ith joint is revolute

(a) The ith joint is revolute

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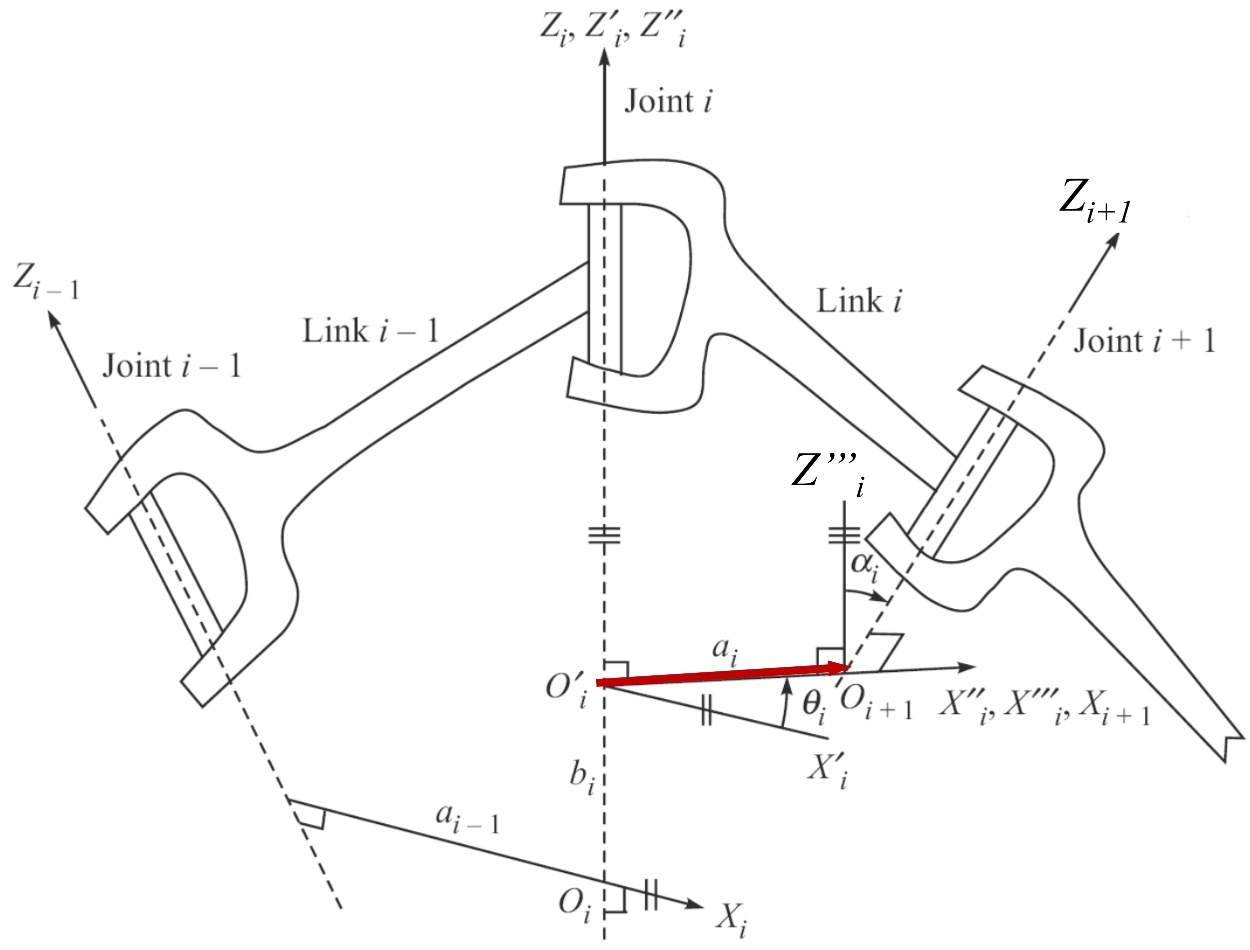
- a_i (Link length): Length between the O'_i and O_{i+1}. This is measured as the distance between the common normals to axes Z_i and Z_{i+1} along X_{i+1}.



(a) The *i*th joint is revolute

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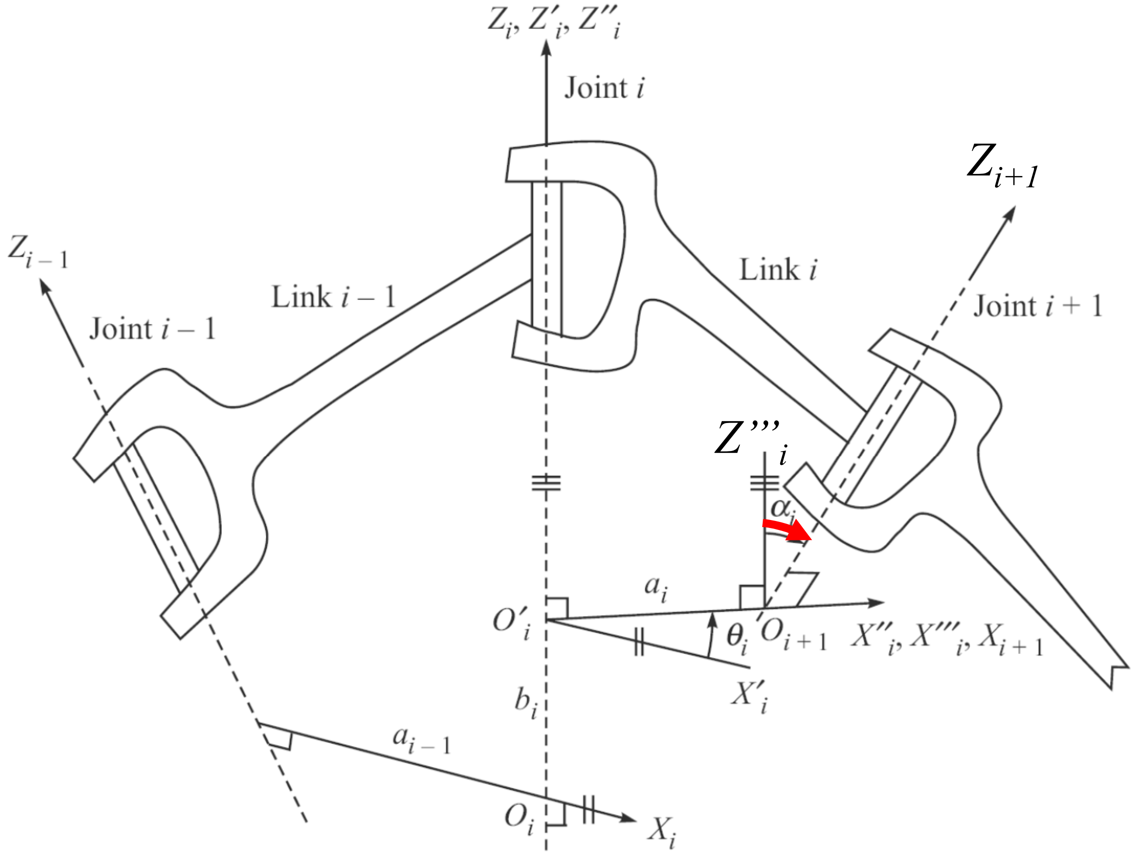
- α_i (Twist angle): Angle between the orthogonal projections of joint axes, Z_i and Z_{i+1} onto a plane normal to the common normal. This is measured as the angle between the axes, Z_i and Z_{i+1}, about axis X_{i+1} to be taken positive when rotation is made counter clockwise.



(a) The ith joint is revolute

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Revolute Joint

- DH@Z (Variable)
  - Joint offset (b)
  - Joint angle (θ)

- DH@X (Const.)
  - Link length (a)
  - Twist angle (α)

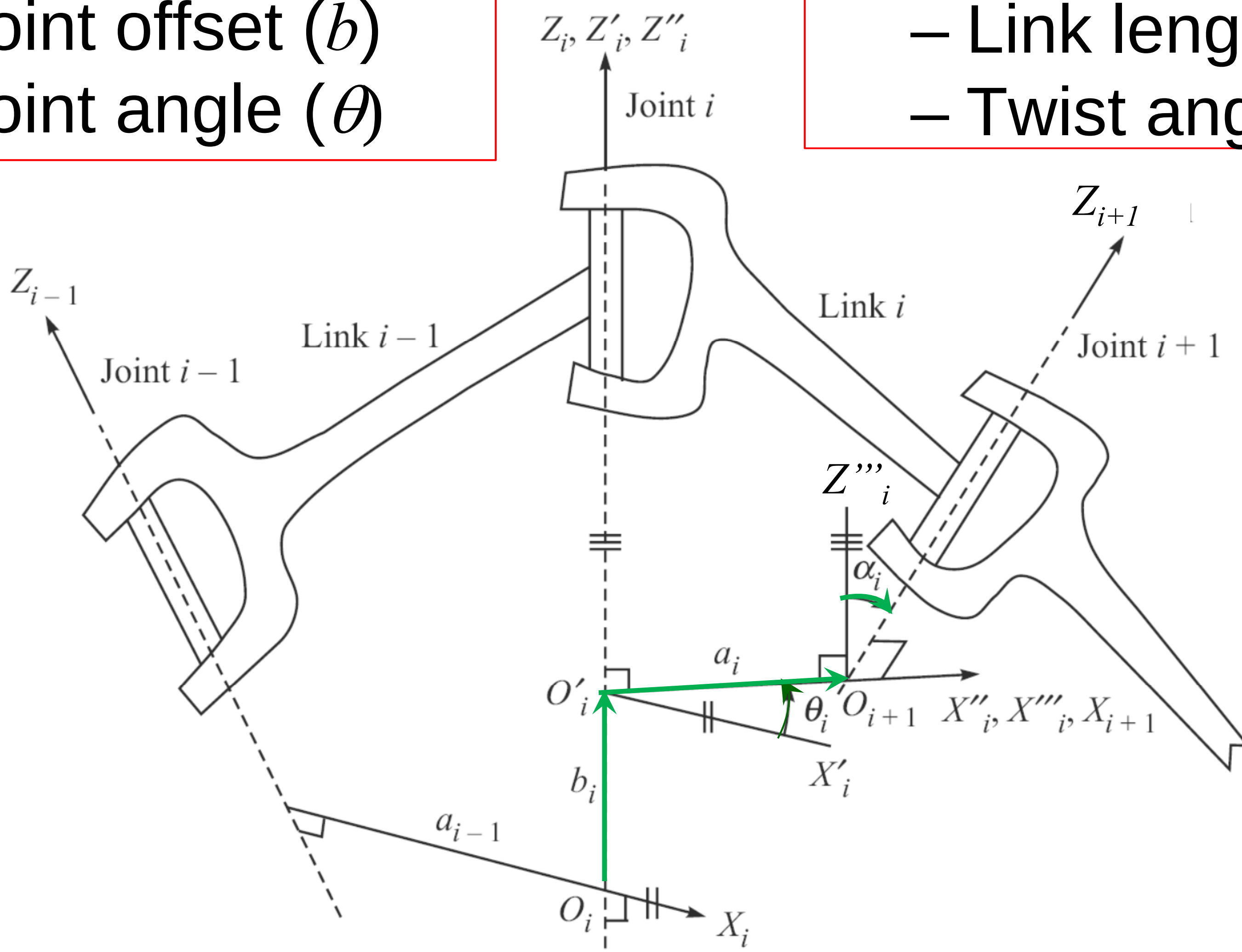

Fig. 5.28 (a) The ith joint is revolute

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Mathematically

- Translation along Z_i

$$\mathbf{T}_b = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & b_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots (5.60a)$$

- Rotation about Z_i

$$\mathbf{T}_\theta = \begin{bmatrix} C\theta_i & -S\theta_i & 0 & 0 \\ S\theta_i & C\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots (5.60b)$$

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- Translation along X_{i+1}

$$\mathbf{T}_a = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \ldots (5.60c)$$

- Rotation about X_{i+1}

$$\mathbf{T}_\alpha = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\alpha_i & -S\alpha_i & 0 \\ 0 & S\alpha_i & C\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \ldots (5.60d)$$

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- Total transformation from Frame *i* to Frame *i+1*

$$\mathbf{T}_i = \mathbf{T}_b \mathbf{T}_\theta \mathbf{T}_a \mathbf{T}_\alpha \qquad \ldots (5.61a)$$

Do it yourself!

$$\mathbf{T}_i = \begin{bmatrix} \multicolumn{3}{c}{\text{Rotation Matrix}} & \text{Position} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\ldots (5.61b)$$

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Spherical-type Arm

- DH-parameters

Link	b_i	θ_i	a_i	α_i
1				
2				
3				

Fill-up the DH parameters

RoboAnalyzer

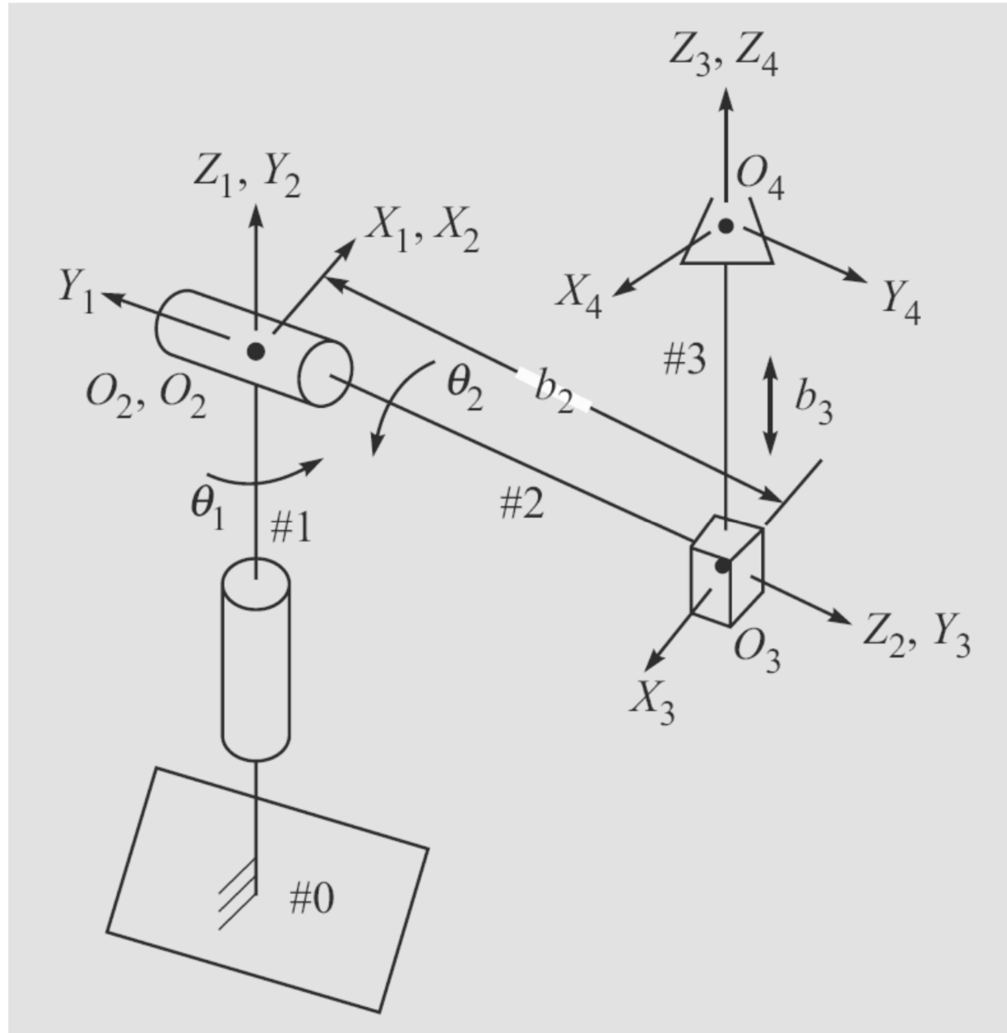

Fig. 5.32 A spherical arm

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PUMA 560

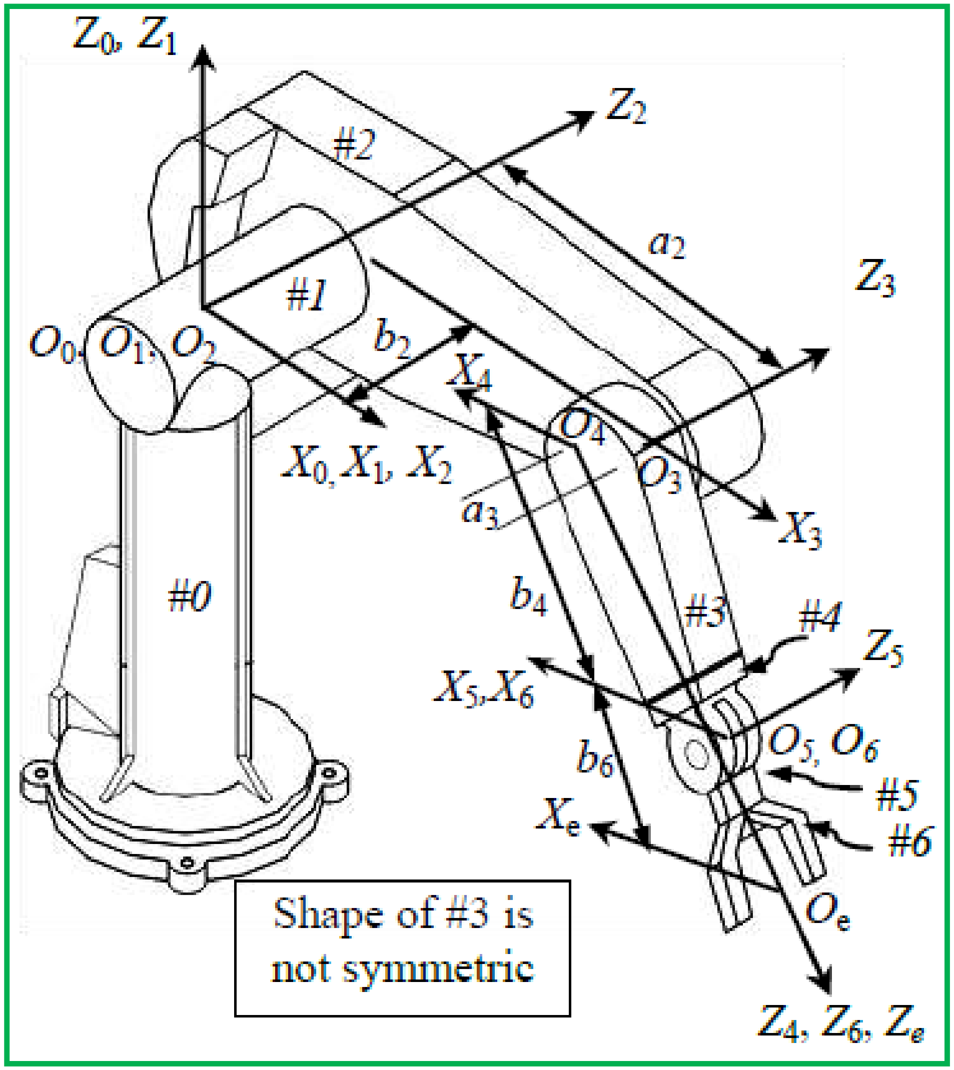

Fig. 5.35 PUMA 560 and its frames

i	Variable DH		Constant DH	
	b_i	θ_i	a_i	α_i
1	0	θ_1	0	$-\pi/2$
2	0	θ_2	a_2	0
3	B_3	θ_3	a_3	$-\pi/2$
4	b_4	θ_4	0	$\pi/2$
5	0	θ_5	0	$-\pi/2$
6	0	θ_6	0	0

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Summary

- Transformation
- DH Paramters
- RoboAnalyzer to visualize

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THANK YOU

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