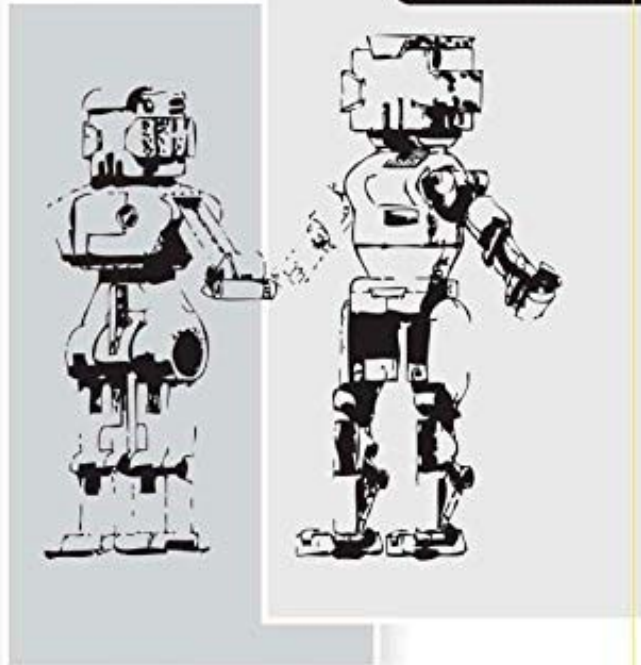


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**Lecture 4**  
**Robot Kinematics (Ch. 6)**

by

**S.K. Saha**

Sep. 05, 2017 (Tu)@JRL301 (Rob. Tech.)

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# Forward and Inverse Kinematics

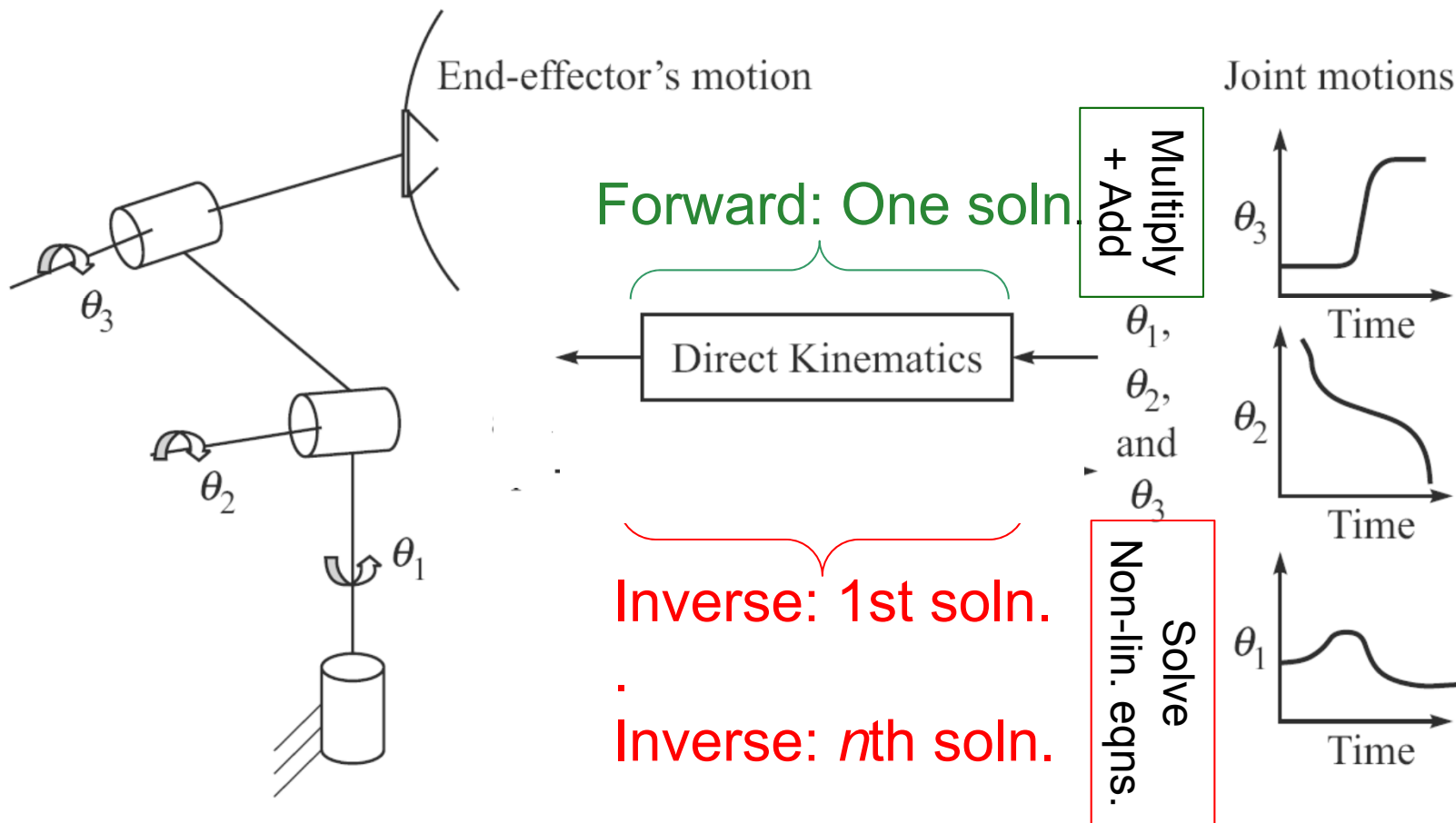


Fig. 6.1 Forward and inverse kinematics

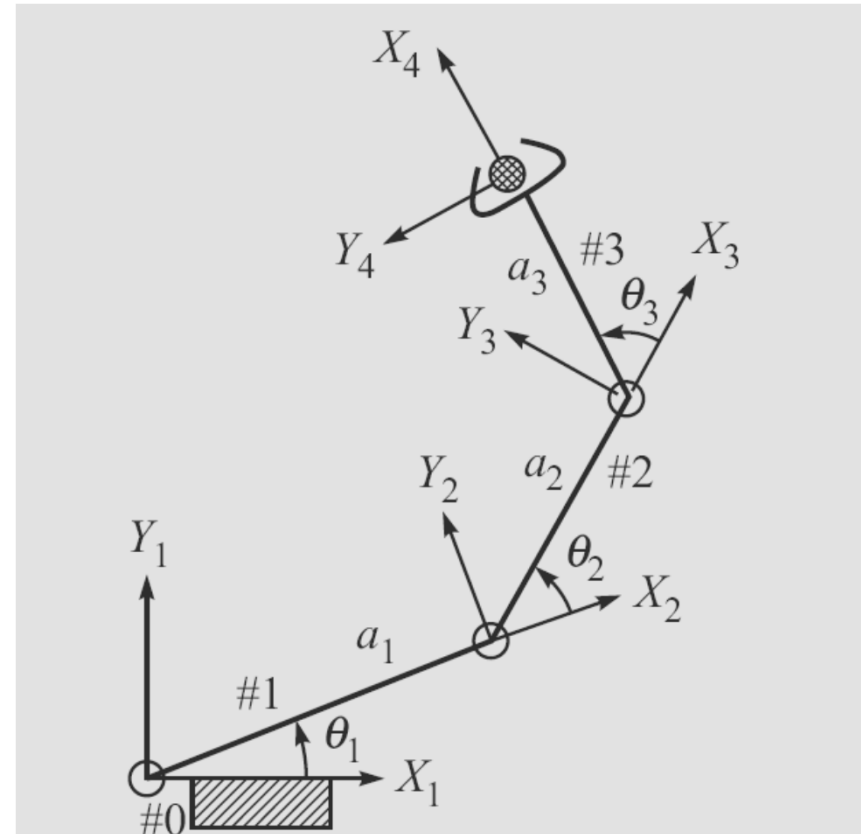
# Three-link Planar Arm

- DH-parameters

Link	$b_i$	$\theta_i$	$a_i$	$\alpha_i$
1	Fill-up the DH parameters			
2				
3				

- Frame transformations (Homogeneous)

$$\mathbf{T}_i = \begin{bmatrix} \text{Fill-up with the elements} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{ for } i=1,2,3$$



**Fig. 5.29** A three-link planar arm

# DH Parameters of Articulated Arm

Link	$b_i$	$\theta_i$	$a_i$	$\alpha_i$
1	0	$\theta_1$ (JV)	0	$-\pi/2$
2	0	$\theta_2$ (JV)	$a_2$	0
3	0	$\theta_3$ (JV)	$a_3$	0

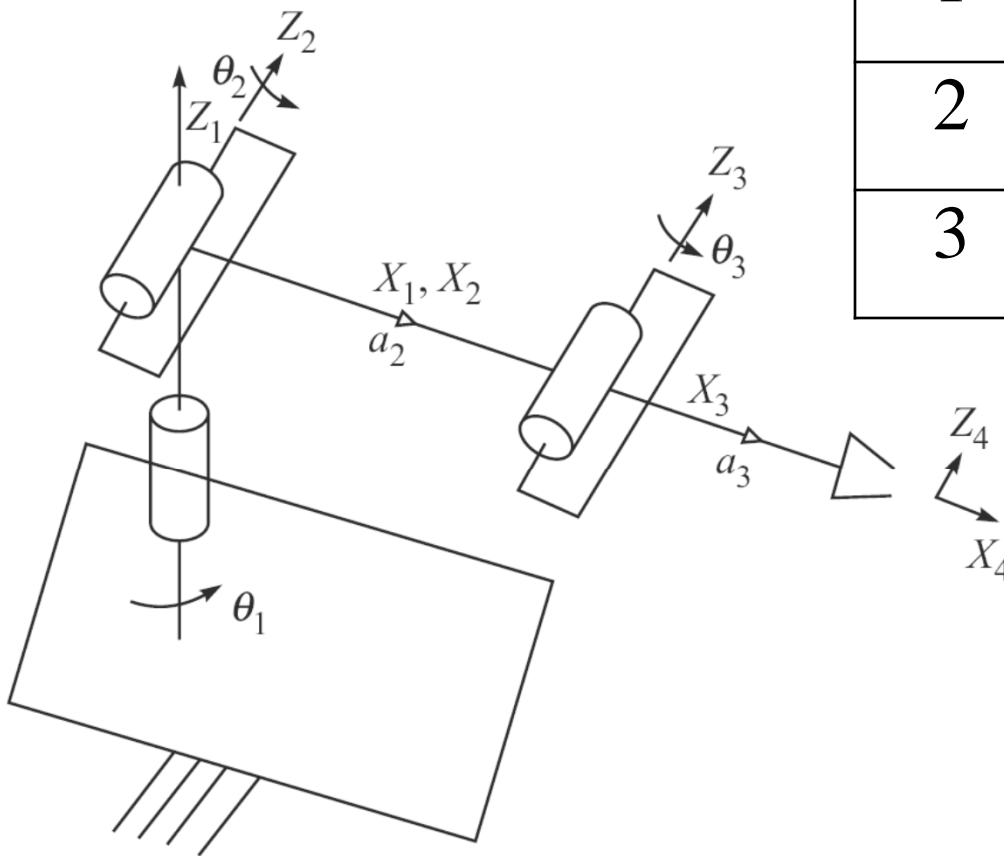


Fig. 5.29 An articulated arm

# Matrices for Articulated Arm

$$\mathbf{T}_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{T}_2 \equiv \begin{bmatrix} c_2 & -s_2 & 0 & a_2 c_2 \\ s_2 & c_2 & 0 & a_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T}_3 \equiv \begin{bmatrix} c_3 & -s_3 & 0 & a_3 c_3 \\ s_3 & c_3 & 0 & a_3 s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

MATLAB

$$\mathbf{T} \equiv \begin{bmatrix} c_1 c_{23} & -c_1 s_{23} & -s_1 & c_1 (a_2 c_2 + a_3 c_{23}) \\ s_1 c_{23} & -s_1 s_{23} & c_1 & s_1 (a_2 c_2 + a_3 c_{23}) \\ -s_{23} & -c_{23} & 0 & -(a_2 s_2 + a_3 s_{23}) \\ 0 & 0 & 0 & 1 \end{bmatrix} \dots (6.11)$$

# Inverse Kinematics

- Unlike Forward Kinematics, general solutions are not possible.
- Several architectures are to be solved differently.

# Two-link Arm

$$p_x = a_1 c_1 + a_2 c_{12}$$

$$p_y = a_1 s_1 + a_2 s_{12}$$

$$c_2 = \frac{p_x^2 + p_y^2 - a_1^2 - a_2^2}{2 a_1 a_2}$$

$$s_2 = \pm \sqrt{1 - c_2^2}$$

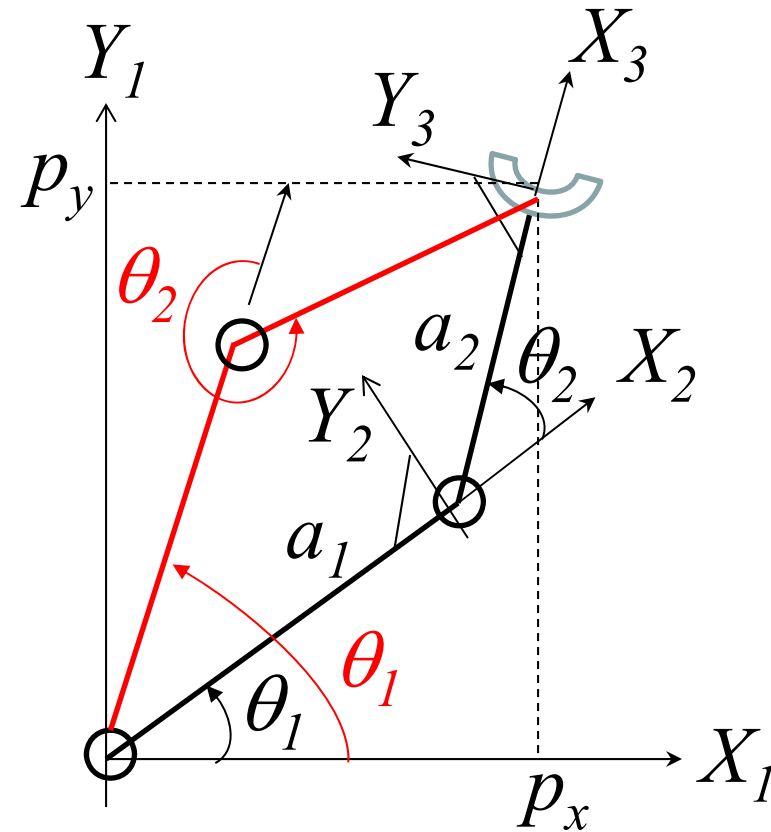
$$\theta_2 = \text{atan2}(s_2, c_2)$$

$$s_1 = \frac{(a_1 + a_2 c_2) p_y - a_2 s_2 p_x}{\Delta}$$

$$c_1 = \frac{(a_1 + a_2 c_2) p_x + a_2 s_2 p_y}{\Delta}$$

$$\Delta \equiv a_1^2 + a_2^2 + 2 a_1 a_2 c_2 = p_x^2 + p_y^2$$

$$\theta_1 = \text{atan2}(s_1, c_1)$$



RoboAnalyzer

# Inverse Kinematics of 3-DOF RRR Arm

$$\varphi = \theta_1 + \theta_2 + \theta_3 \dots (6.18a)$$

$$p_x = a_1 c_1 + a_2 c_{12} + a_3 c_{123} \dots (6.18b)$$

$$p_y = a_1 s_1 + a_2 s_{12} + a_3 s_{123} \dots (6.18c)$$

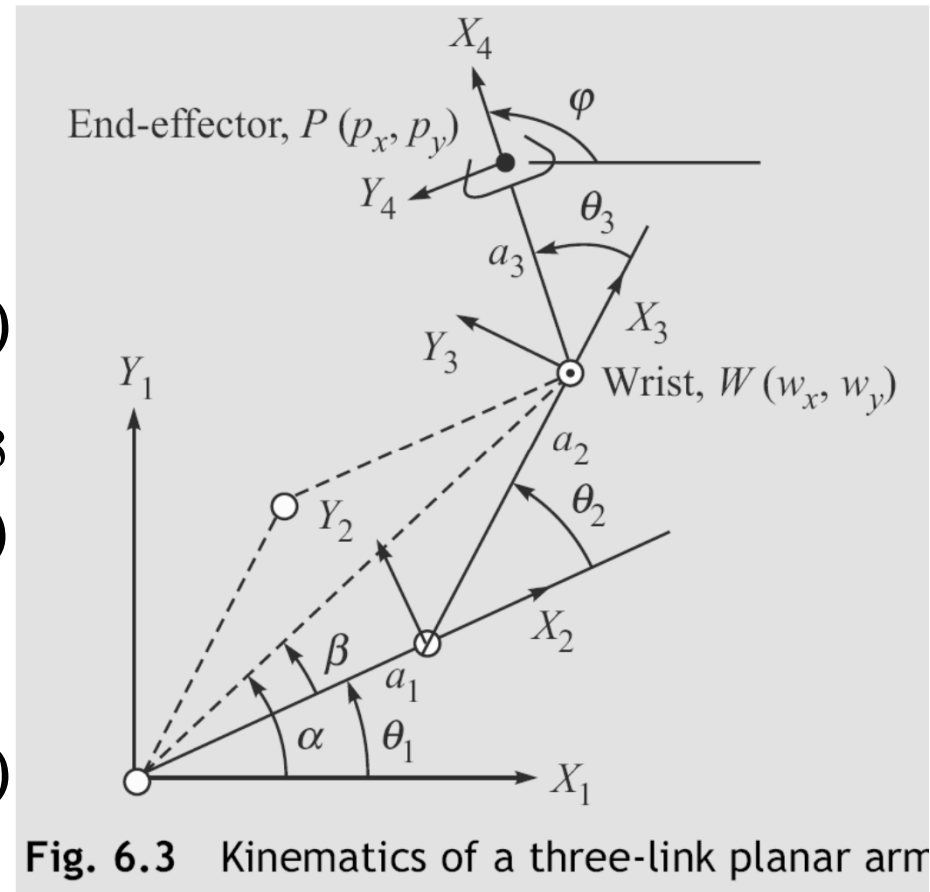


Fig. 6.3 Kinematics of a three-link planar arm

$$w_x = p_x - a_3 c \varphi = a_1 c_1 + a_2 c_{12} \dots (6.19a)$$

$$w_y = p_y - a_3 s \varphi = a_1 s_1 + a_2 s_{12} \dots (6.19b)$$



$$w_x^2 + w_y^2 = a_1^2 + a_2^2 + 2 a_1 a_2 c_2 \quad \dots (6.20a)$$

$$c_2 = \frac{w_x^2 + w_y^2 - a_1^2 - a_2^2}{2 a_1 a_2} \quad s_2 = \pm \sqrt{1 - c_2^2} \quad \dots (6.20b,c)$$

$$\theta_2 = \text{atan2}(s_2, c_2) \quad \dots (6.21)$$

$$w_x = (a_1 + a_2 c_2) c_1 - a_2 s_1 s_2 \quad \dots (6.22a)$$

$$w_y = (a_1 + a_2 c_2) s_1 + a_2 c_1 s_2 \quad \dots (6.22b)$$

$$s_1 = \frac{(a_1 + a_2 c_2) w_y - a_2 s_2 w_x}{\Delta} \quad c_1 = \frac{(a_1 + a_2 c_2) w_x + a_2 s_2 w_y}{\Delta} \quad \dots (6.23a,b)$$

$$\Delta \equiv a_1^2 + a_2^2 + 2 a_1 a_2 c_2 = w_x^2 + w_y^2$$

$$\theta_1 = \text{atan2}(s_1, c_1) \quad \dots (6.23c)$$

$$\theta_3 = \varphi - \theta_1 - \theta_2 \quad \dots (6.24)$$

# Numerical Example

- An RRR planar arm (Example 6.15). Input

$$\mathbf{T} \equiv \begin{bmatrix} \text{Rotation Matrix} & \text{Origin of end-effector frame} & 4.23 \\ 0 & 0 & 0 & 1 & 1.86 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

where  $\phi = 60^\circ$ , and  $a_1 = a_2 = 2$  units, and  $a_3 = 1$  unit.

Do it yourself → Verify using [RoboAnalyzer](#)

Using eqs. (6.13b-c),

$$c_2 = 0.866, \text{ and } s_2 = 0.5,$$

$$\theta_2 = 30^\circ$$

Next, from eqs. (6.16a-b),

$$s_1 = 0, \text{ and } c_1 = 0.866.$$

$$\theta_1 = 0^\circ.$$

Finally, from eq. (6.17),

$$\theta_3 = 30^\circ.$$

Therefore

$$\theta_1 = 0^\circ \theta_2 = 30^\circ, \text{ and } \theta_3 = 30^\circ$$

...(6.30b)

The positive values of  $s_2$  was used in evaluating  $\theta_2 = 30^\circ$ .

The use of negative value would result in :

$$\theta_1 = 30^\circ \theta_2 = -30^\circ, \text{ and } \theta_3 = 60^\circ$$

...(6.30c)

**MATLAB**  
program

# Watch

- Forward and Inverse Kinematics: Watch 3/3 of IGNOU Lectures [29min]

<https://www.youtube.com/watch?v=duKD8cvtBTI>

- For more clarity: Watch 12 of Addis Ababa Lectures [77 min]

[\[https://www.youtube.com/watch?v=NXWzk1toze4\]](https://www.youtube.com/watch?v=NXWzk1toze4)

- Robotics (13 of Addis Ababa Lectures): Inverse Kinematics [82 min]

<https://www.youtube.com/watch?v=uIP3YiJLiEM>

# Summary

- Forward Kinematics
- Inverse kinematics
  - A spatial 6-DOF wrist-portioned has 8 solutions

THANK YOU

saha@mech.iitd.ac.in

<http://sksaha.com>