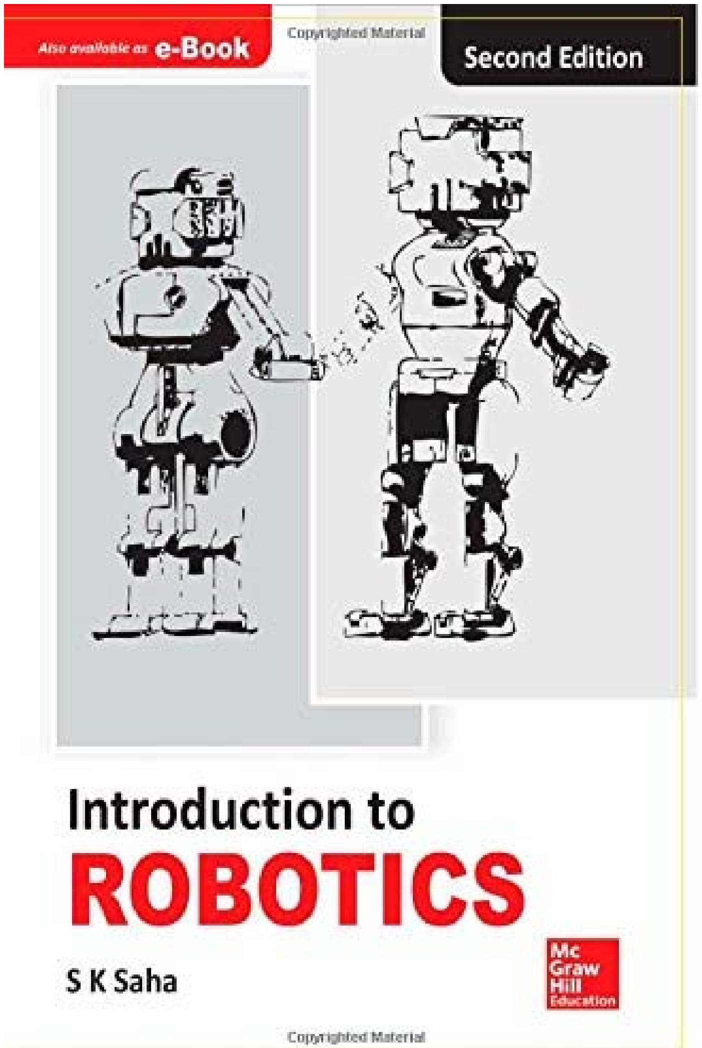
Mc Graw Hill Education



Lecture 4

Robot Kinematics (Ch. 6)

by

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Forward and Inverse Kinematics

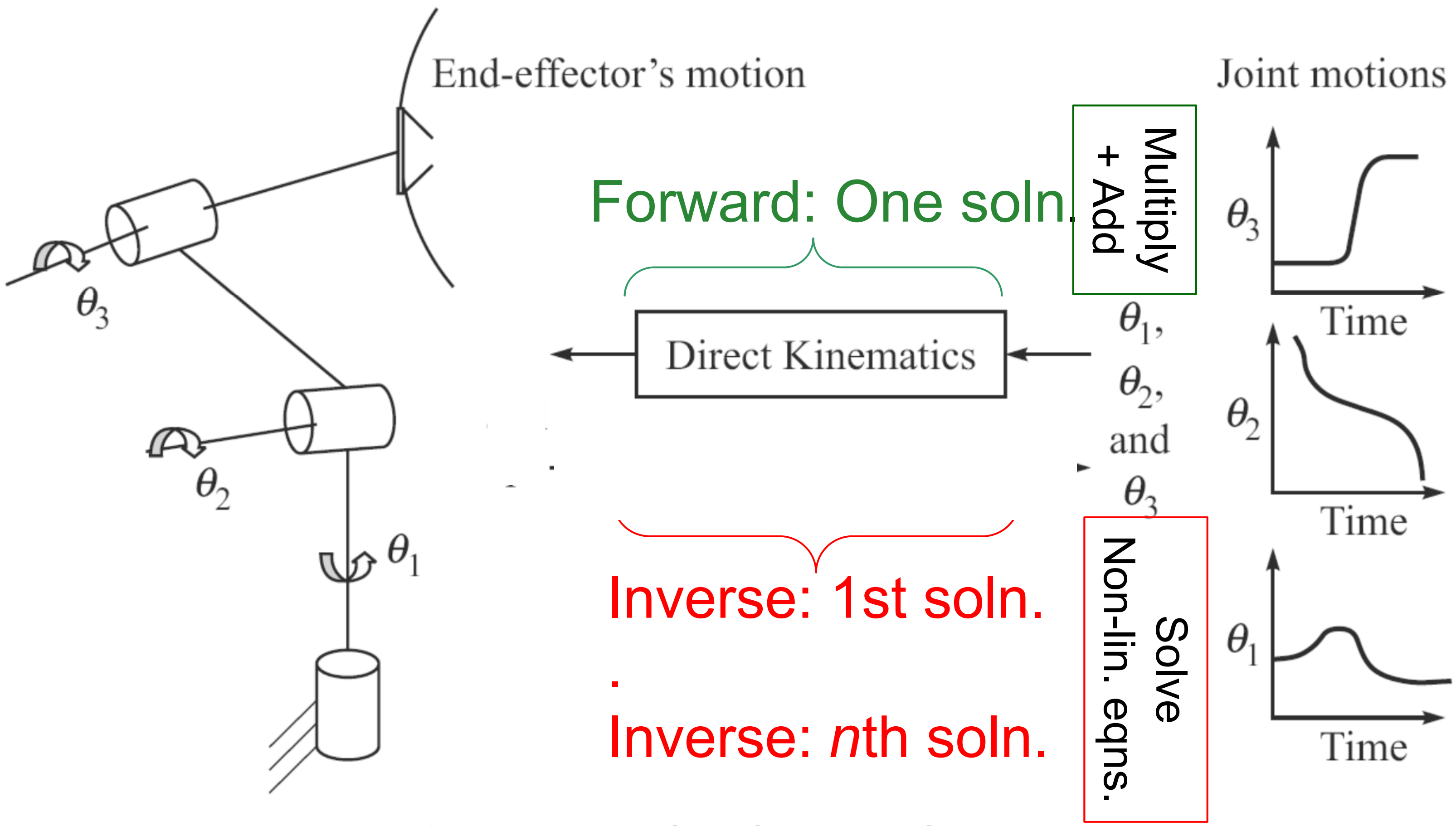


Fig. 6.1 Forward and inverse kinematics

Three-link Planar Arm

- DH-parameters

Link	b_i	θ_i	a_i	α_i
1				
2		Fill-up the DH parameters		
3				

- Frame transformations (Homogeneous)

$$\mathbf{T}_i = \begin{bmatrix} \text{Fill-up with the elements} & & & \\ & & & \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{ for } i=1,2,3$$

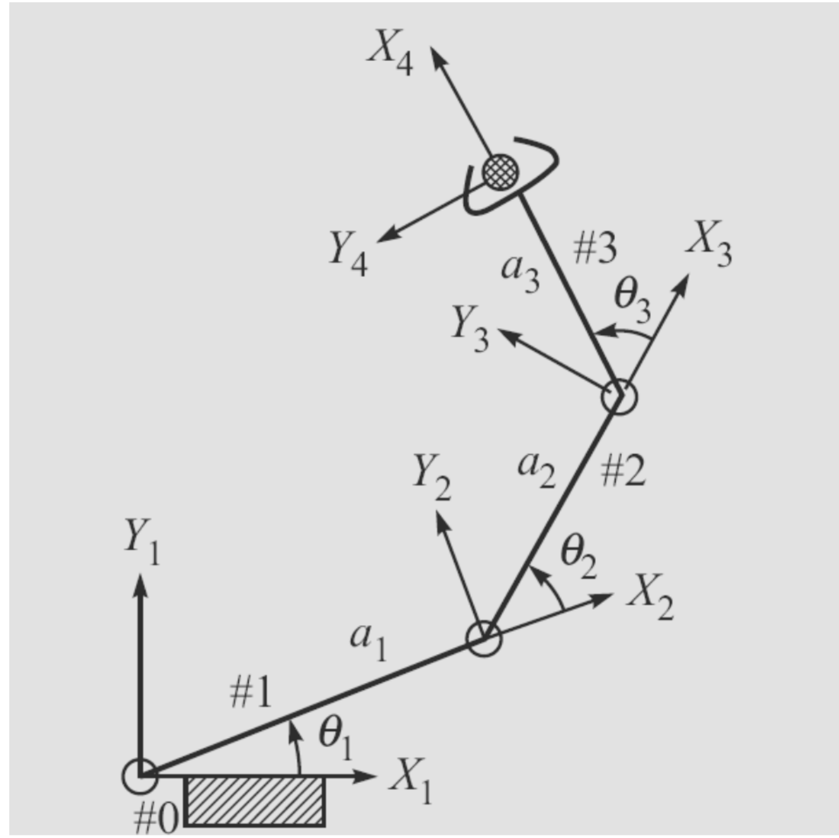


Fig. 5.29 A three-link planar arm

DH Parameters of Articulated Arm

Link	b_i	θ_i	a_i	α_i
1	0	θ_1 (JV)	0	$-\pi/2$
2	0	θ_2 (JV)	a_2	0
3	0	θ_3 (JV)	a_3	0

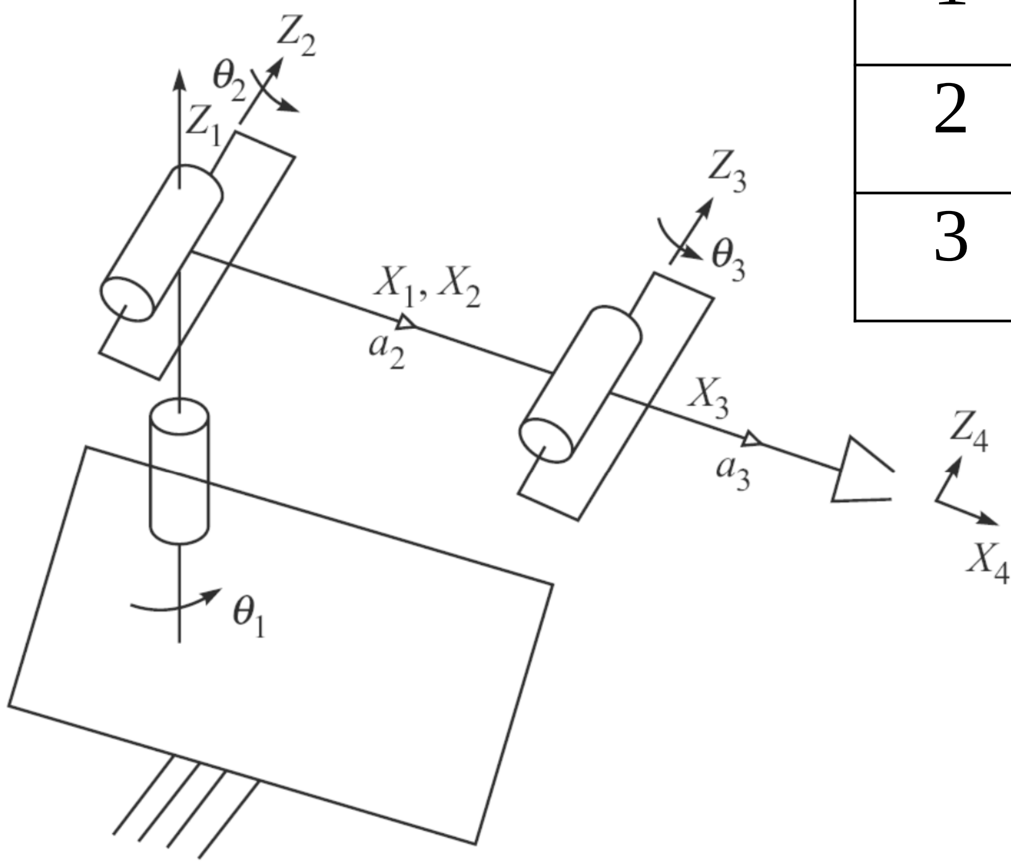


Fig. 5.29 An articulated arm

Matrices for Articulated Arm

$$\mathbf{T}_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{T}_2 \equiv \begin{bmatrix} c_2 & -s_2 & 0 & a_2 c_2 \\ s_2 & c_2 & 0 & a_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T}_3 \equiv \begin{bmatrix} c_3 & -s_3 & 0 & a_3 c_3 \\ s_3 & c_3 & 0 & a_3 s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

MATLAB

$$\mathbf{T} \equiv \begin{bmatrix} c_1 c_{23} & -c_1 s_{23} & -s_1 & c_1 (a_2 c_2 + a_3 c_{23}) \\ s_1 c_{23} & -s_1 s_{23} & c_1 & s_1 (a_2 c_2 + a_3 c_{23}) \\ -s_{23} & -c_{23} & 0 & -(a_2 s_2 + a_3 s_{23}) \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots (6.11)$$

Inverse Kinematics

- Unlike Forward Kinematics, general solutions are not possible.
- Several architectures are to be solved differently.

Two-link Arm

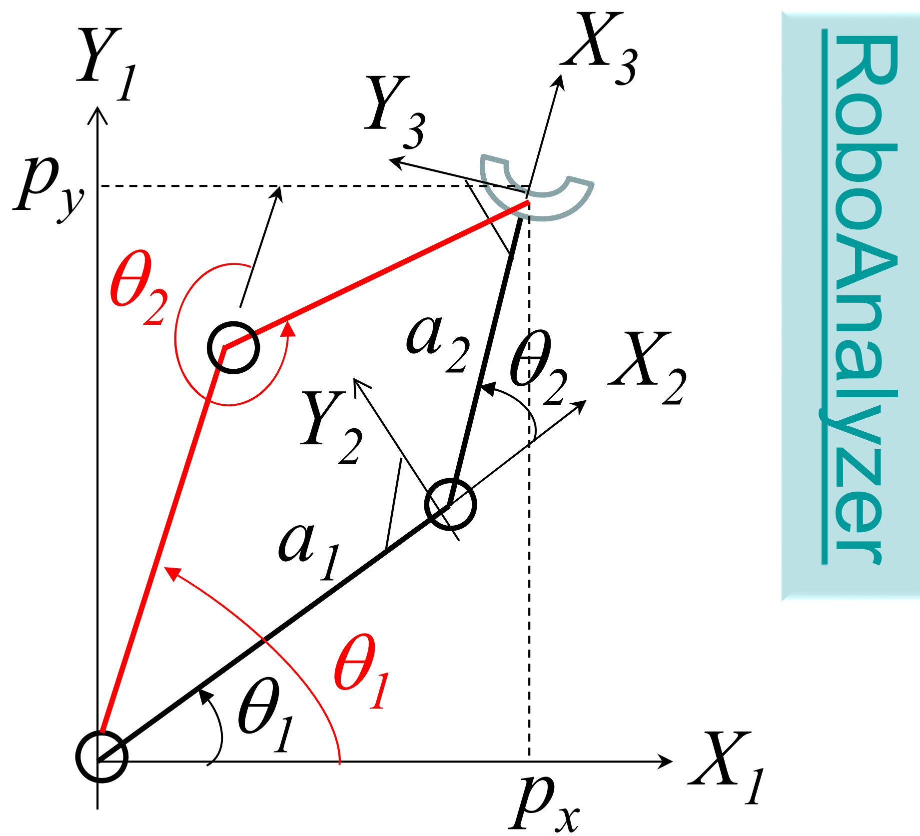
$$p_x = a_1 c_1 + a_2 c_{12}$$

$$p_y = a_1 s_1 + a_2 s_{12}$$

$$c_2 = \frac{p_x^2 + p_y^2 - a_1^2 - a_2^2}{2 a_1 a_2}$$

$$s_2 = \pm\sqrt{1 - c_2^2}$$

$$\theta_2 = \text{atan2}(s_2, c_2)$$



$$s_1 = \frac{(a_1 + a_2 c_2) p_y - a_2 s_2 p_x}{\Delta}$$

$$\Delta \equiv a_1^2 + a_2^2 + 2 a_1 a_2 c_2 = p_x^2 + p_y^2$$

$$c_1 = \frac{(a_1 + a_2 c_2) p_x + a_2 s_2 p_y}{\Delta}$$

$$\theta_1 = \text{atan2}(s_1, c_1)$$

Inverse Kinematics of 3-DOF RRR Arm

$$\varphi = \theta_1 + \theta_2 + \theta_3 \quad \ldots (6.18a)$$

$$p_x = a_1 c_1 + a_2 c_{12} + a_3 c_{123} \quad \ldots (6.18b)$$

$$p_y = a_1 s_1 + a_2 s_{12} + a_3 s_{123} \quad \ldots (6.18c)$$

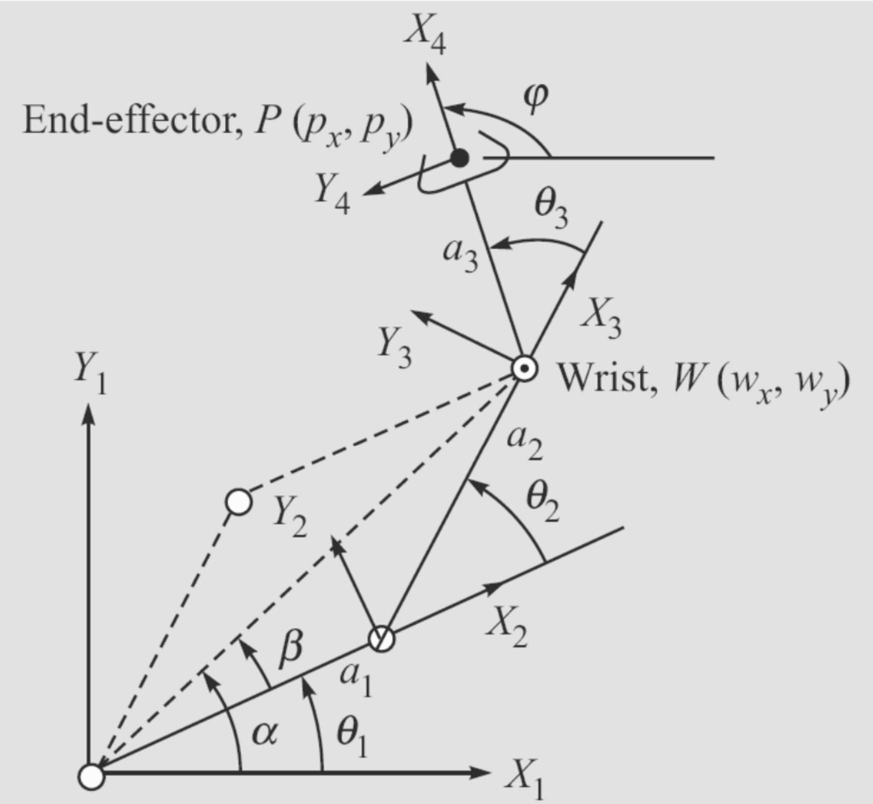


Fig. 6.3 Kinematics of a three-link planar arm

$$w_x = p_x - a_3 c\,\varphi = a_1 c_1 + a_2 c_{12} \quad \ldots (6.19a)$$

$$w_y = p_y - a_3 s\,\varphi = a_1 s_1 + a_2 s_{12} \quad \ldots (6.19b)$$

$$w^2_x + w^2_y = a_1^2 + a_2^2 + 2\, a_1 a_2 c_2 \qquad \ldots (6.20a)$$

$$c_2 = \frac{w_1^2 + w_2^2 - a_1^2 - a_2^2}{2\, a_1\, a_2} \quad s_2 = \pm\sqrt{1 - c_2^2} \qquad \ldots (6.20b,c)$$

$$\theta_2 = \text{atan2}\,(s_2, c_2) \qquad \ldots (6.21)$$

$$w_x = (a_1 + a_2 c_2) c_1 - a_2 s_1 s_2 \qquad \ldots (6.22a)$$

$$w_y = (a_1 + a_2 c_2) s_1 + a_2 c_1 s_2 \qquad \ldots (6.22b)$$

$$s_1 = \frac{(a_1 + a_2 c_2) w_y - a_2 s_2 w_x}{\Delta} \quad c_1 = \frac{(a_1 + a_2 c_2) w_x + a_2 s_2 w_y}{\Delta}$$

$$\Delta \equiv a_1^2 + a_2^2 + 2 a_1 a_2 c_2 = w_x^2 + w_y^2 \qquad \ldots (6.23a,b)$$

$$\theta_1 = \text{atan2}\,(s_1, c_1) \qquad \ldots (6.23c)$$

$$\theta_3 = \varphi - \theta_1 - \theta_2 \qquad \ldots (6.24)$$

Numerical Example

- An RRR planar arm (Example 6.15). Input

T ≡

Rotation Matrix

Origin of end-effector frame

4.23

1.86

0

$0 \quad 0 \quad 0 \quad 1$

where $\phi = 60^o$, and $a_1 = a_2 = 2$ units, and $a_3 = 1$ unit.

Do it yourself → Verify using RoboAnalyzer

Using eqs. (6.13b-c), $c_2 = 0.866$, and $s_2 = 0.5$,

$\theta_2 = 30^o$

Next, from eqs. (6.16a-b), $s_1 = 0$, and $c_1 = 0.866$.

$\theta_1 = 0^o$.

MATLAB program

Finally, from eq. (6.17) ,

$\theta_3 = 30^o$.

Therefore $\theta_1 = 0^o$ $\theta_2 = 30^o$, and $\theta_3 = 30$ …(6.30b)

The positive values of s_2 was used in evaluating $\theta_2 = 30^o$.

The use of negative value would result in :

$\theta_1 = 30^o$ $\theta_2 = -30^o$, and $\theta_3 = 60^o$ …(6.30c)

Watch

- Forward and Inverse Kinematics: Watch 3/3 of IGNOU Lectures [29min]

https://www.youtube.com/watch?v=duKD8cvtBTI

- For more clarity: Watch 12 of Addis Ababa Lectures [77 min]

[https://www.youtube.com/watch?v=NXWzk1toze4

- Robotics (13 of Addis Ababa Lectures): Inverse Kinematics [82 min]

https://www.youtube.com/watch?v=uIP3YiJLiEM

Summary

- Forward Kinematics
- Inverse kinematics
  - A spatial 6-DOF wrist-portioned has 8 solutions

THANK YOU

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