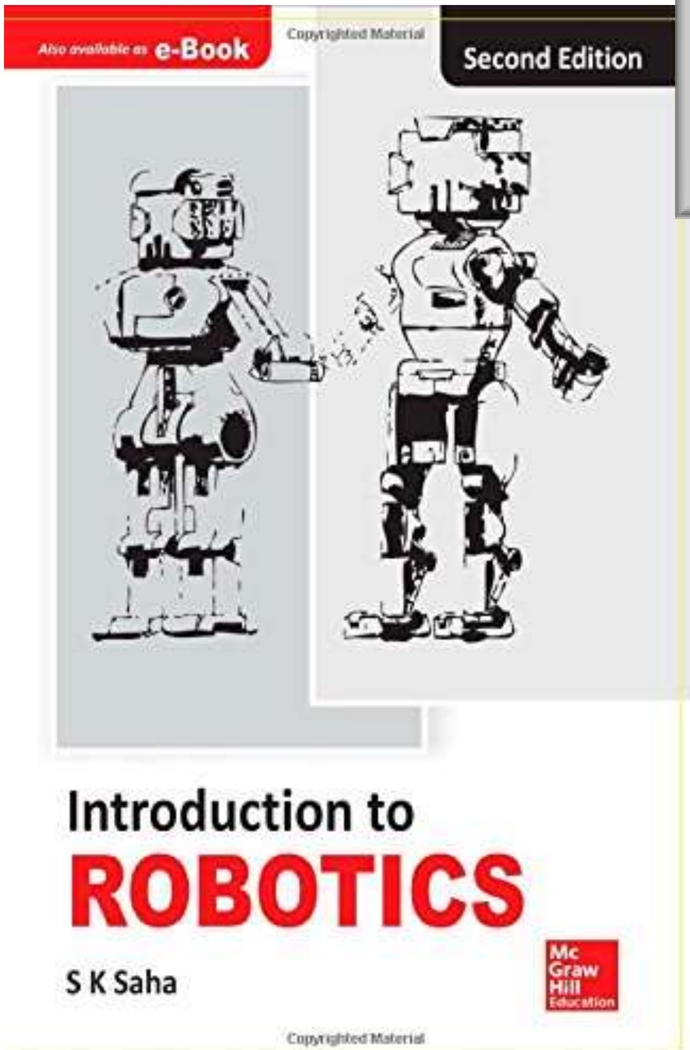
McGraw-Hill Education



Lecture 5 (SIT Sem. Rm.)

by

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7

Statics and Manipulator Design

Principle of Virtual Work

$$\mathbf{w}_e^T \delta\mathbf{x} = \boldsymbol{\tau}^T \delta\boldsymbol{\theta} \qquad \ldots (7.28)$$

- Relation between two virtual displacements (Can be derived from velocity expression)

$$\delta\mathbf{x} = \mathbf{J}\delta\boldsymbol{\theta} \qquad \ldots (7.29)$$

$$\mathbf{w}_e^T \mathbf{J}\delta\boldsymbol{\theta} = \boldsymbol{\tau}^T \delta\boldsymbol{\theta} \Rightarrow \mathbf{w}_e^T \mathbf{J} = \boldsymbol{\tau}^T \qquad \ldots (7.31)$$

$$\boxed{\boldsymbol{\tau} = \mathbf{J}^T \mathbf{w}_e} \qquad \ldots (7.32)$$

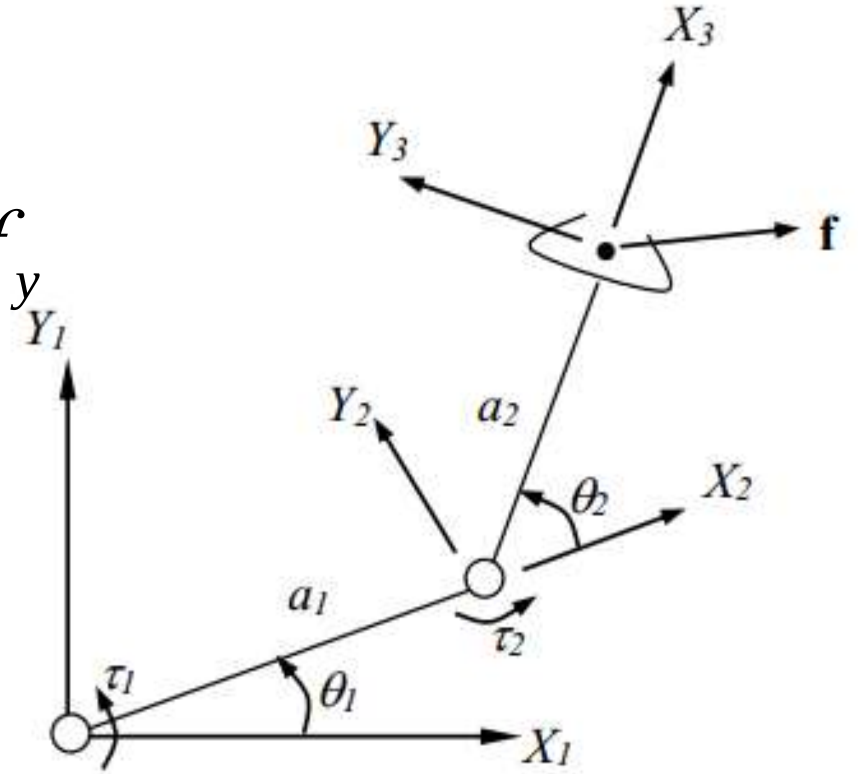
Example: 2-link RR Planar Arm

From FBD

$$\tau_1 = [\mathbf{e}_1]_1^T [\mathbf{n}_{01}]_1$$

$$= a_1 f_x s\theta_2 + (a_2 + a_1 c\theta_2) f_y$$

$$\tau_2 = [\mathbf{e}_2]_2^T [\mathbf{n}_{12}]_2 = a_2 f_y$$



$$\boxed{\boldsymbol{\tau} = \mathbf{J}^{\mathrm{T}} \mathbf{f}}$$

$$\boldsymbol{\tau} \equiv \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \quad \mathbf{J}^T \equiv \begin{bmatrix} a_1 s\theta_2 & a_1 c\theta_2 + a_2 & 0 \\ 0 & a_2 & 0 \end{bmatrix} \quad \mathbf{f} \equiv \begin{bmatrix} f_x \\ f_y \\ 0 \end{bmatrix}$$

Two Jacobian Matrices

- From Statics

$$\mathbf{J} \equiv \begin{bmatrix} a_1 s\theta_2 & 0 \\ a_1 c\theta_2 + a_2 & a_2 \\ 0 & 0 \end{bmatrix}$$

- From Kinematics

$$\mathbf{J} \equiv \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \end{bmatrix}$$

Jacobian from Statics in Frame 1

$$[\mathbf{J}]_1 \equiv \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 \\ s\theta_2 & c\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_1 s\theta_2 & 0 \\ a_1 c\theta_2 + a_2 & a_2 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -a_1 s\theta_1 - a_2 s\theta_{12} & -a_2 s\theta_{12} \\ a_1 c\theta_1 + a_2 c\theta_{12} & a_2 c\theta_{12} \\ 0 & 0 \end{bmatrix} \quad \ldots (7.34)$$

- Without the last row, it is the same as the one from kinematics ← Should be!

Manipulator Design

- High investment in robot usage → low technological level of mechanical structure
- Functional Requirements
- Kinetostatic Measures
- Structural Design and Dynamics
- Economics

Functional Requirements of a Robot

- Payload
- Mobility
- Configuration

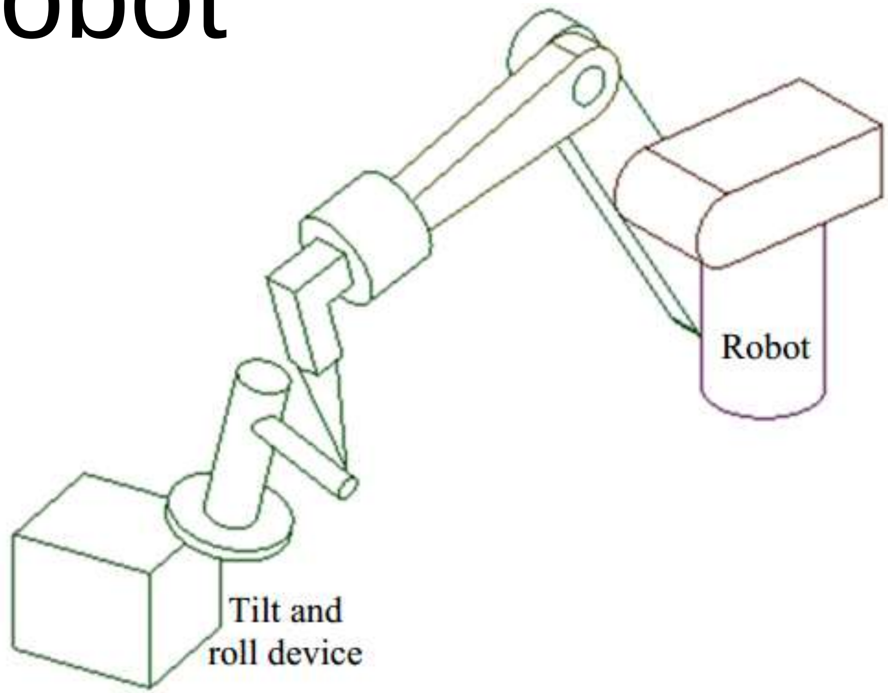


Figure 7.6 A tilt and roll device provides additional DOF to the robot system

- Speed, Accuracy and Repeatability
- Actuators and Sensors

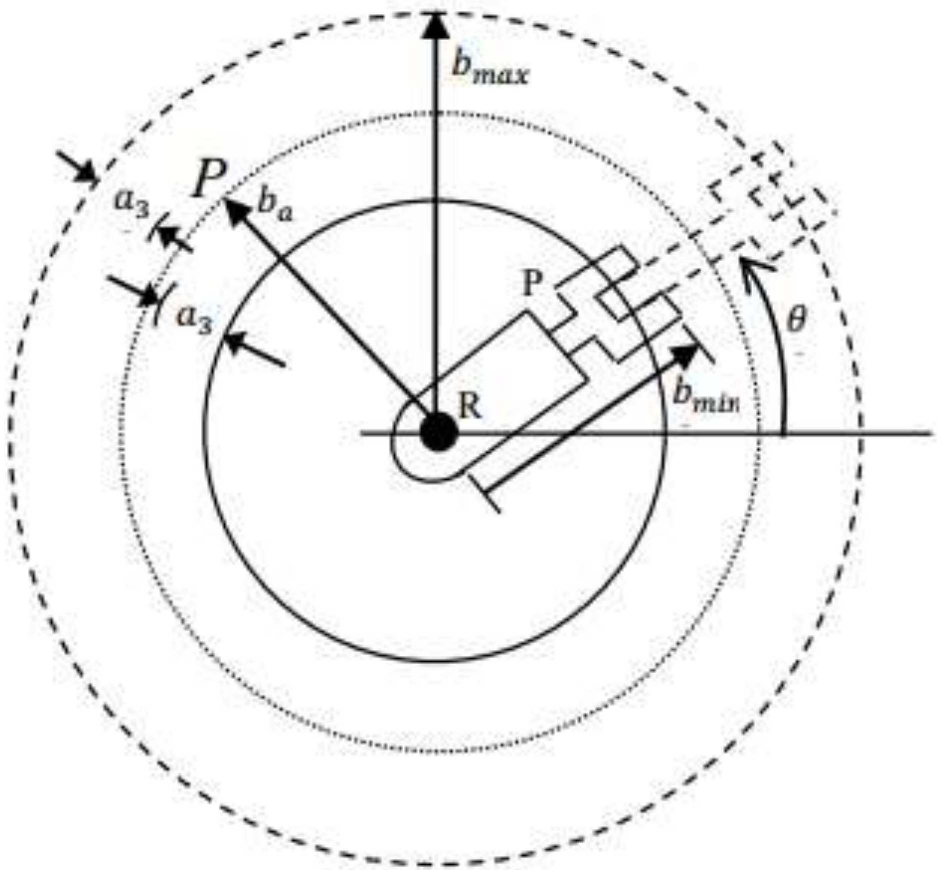


Figure 7.7 Workspace of a 2-DOF RP planar manipulator

$$b_{\min} \le b \le b_{\max}, \text{ for } 0^o \le \theta \le 360^o$$

Dexterity and Manipulability

- Dexterity → $w_d = \det(\mathbf{J})$ … (7.44)
- Manipulability → $w_m = \sqrt{\det(\mathbf{J}\mathbf{J}^T)}$
- Non-redundant manipulator → square Jacobian

$$w_m = \left|\det(\mathbf{J})\right| \qquad w_d = w_m$$

Motor Selection (Thumb Rule)

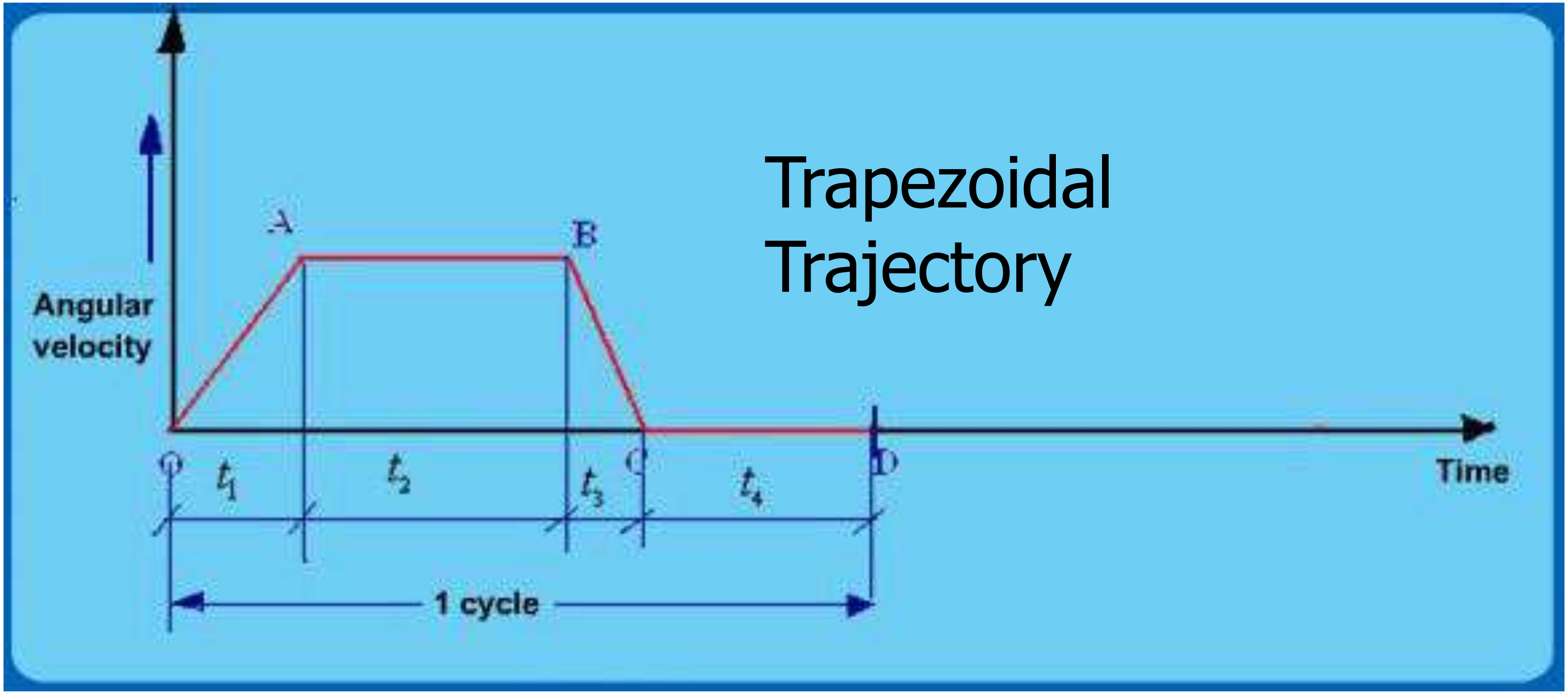
- Rapid movement with high torques (> 3.5 kW): Hydraulic actuator
- < 1.5 kW (no fire hazard): Electric motors
- 1-5 kW: Availability or cost will determine the choice

Simple Calculation

2 m robot arm to lift 25 kg mass at 10 rpm

- Force = 25 x 9.81 = 245.25 N
- Torque = 245.25 x 2 = 490.5 Nm
- Speed = 2π x 10/60 = 1.047 rad/sec
- Power = Torque x Speed = 0.513 kW
- Simple but sufficient for approximation

Practical Application



Subscript l for load; m for motor;

$G = \omega_l/\omega_m\,(< 1)$; η: Motor + Gear box efficiency

Accelerations & Torques

Ang. accn. during t_1: $\alpha_1 = \dfrac{\omega_2 - 0}{t_1}$

Ang. accn. during t_2: Zero (Const. Vel.)

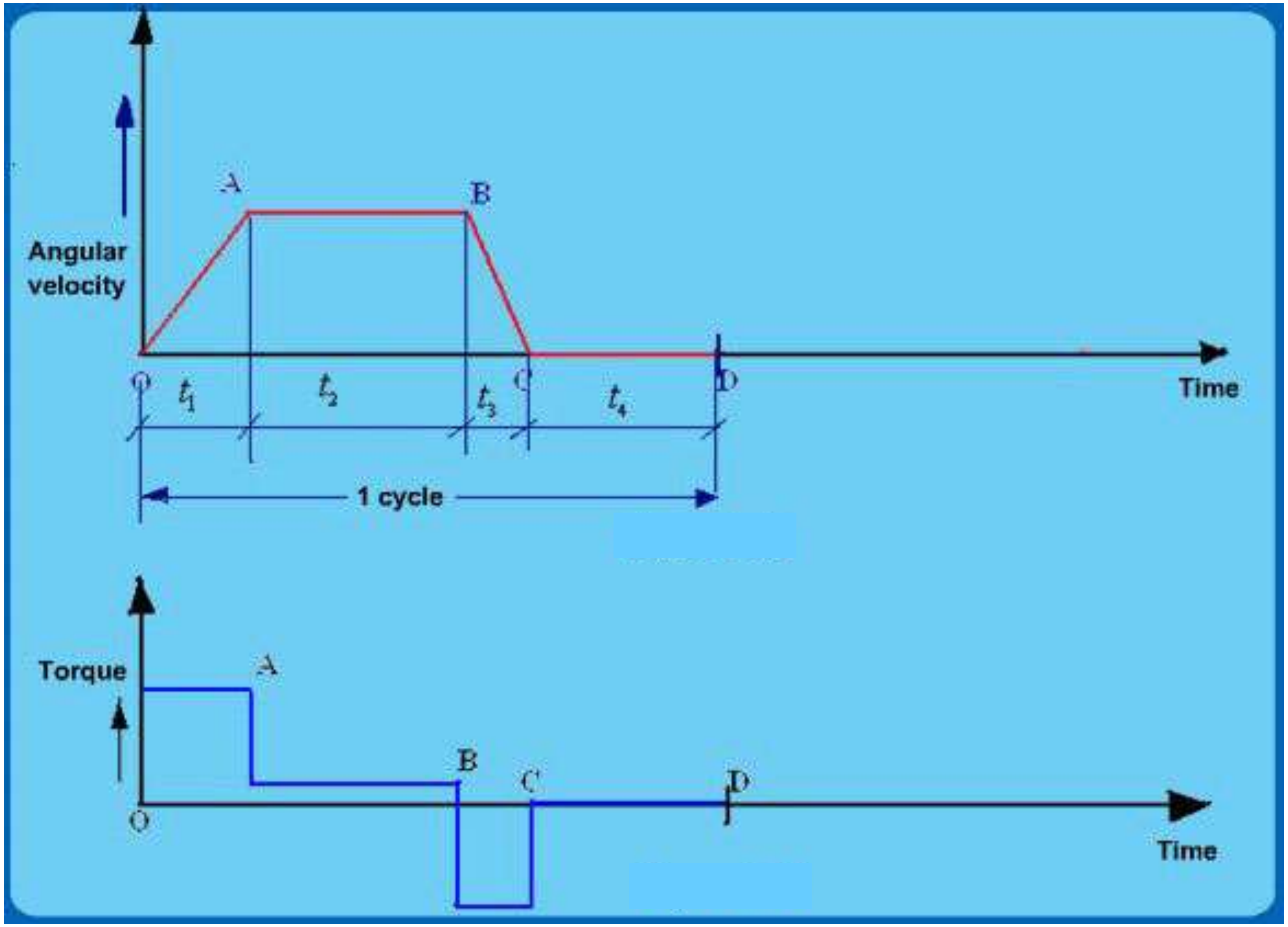
Ang. accn. during t_3: $\alpha_3 = \dfrac{\omega_b - 0}{t_3}$

Torque during t_1: $T_1 =$ $\left(I_m + \dfrac{G^2}{\eta} I_l\right)\alpha_1 + T_f \dfrac{G}{\eta}$

Torque during t_2: $T_2 =$ $T_f \dfrac{G}{\eta}$

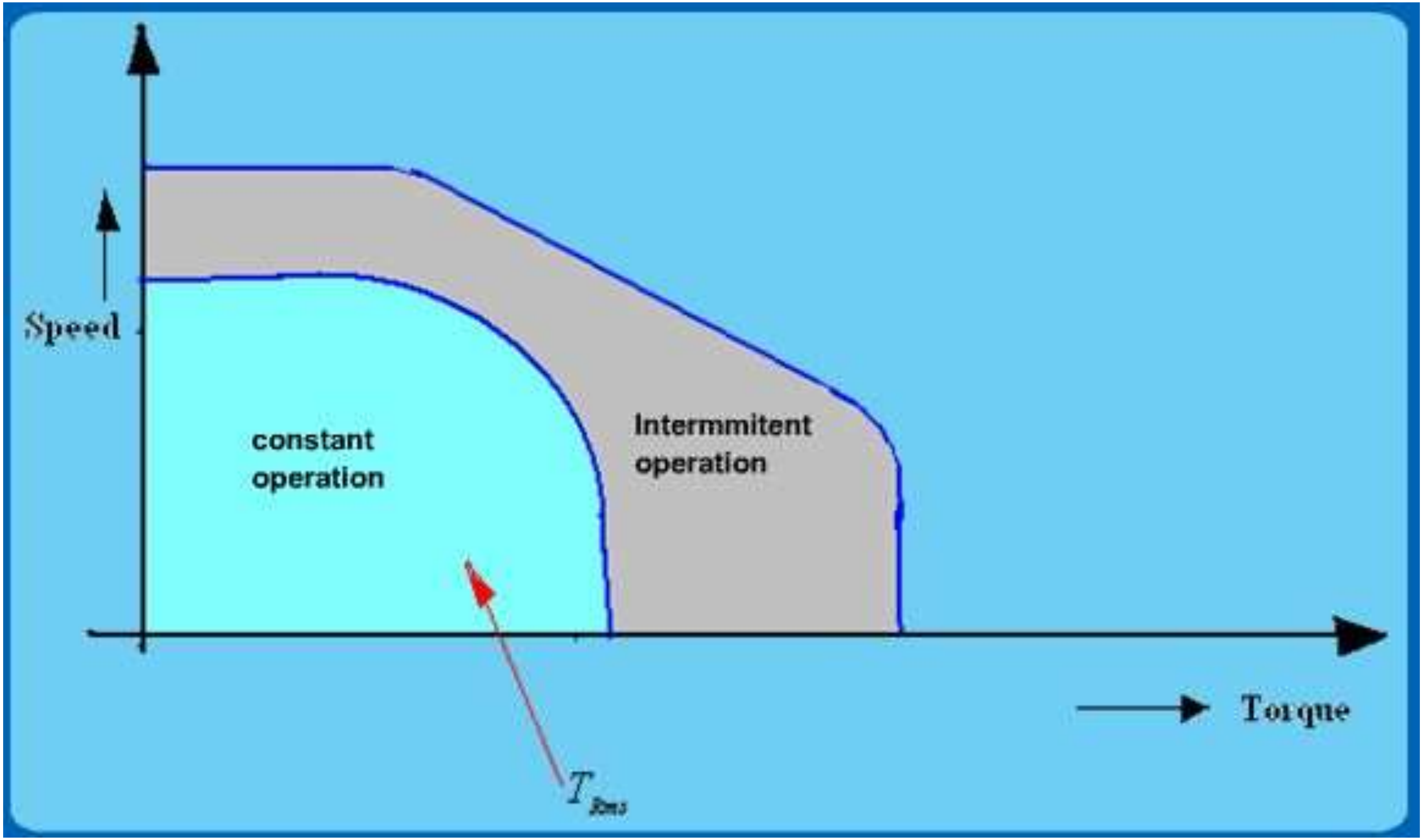
Torque during t_3: $T_3 =$ $\left(I_m + \dfrac{G^2}{\eta} I_l\right)\alpha_3 - T_f \dfrac{G}{\eta}$

RMS Value



$$T_{Rms} = \sqrt{\frac{(T_1^2 \times t_1) + (T_2^2 \times t_2) + (T_3^2 \times t_3) + (zero)t_4}{t_1 + t_2 + t_3 + t_4}}$$

Motor Performance



Final Selection

- Peak speed and peak torque requirements , where T_{Peak} is max of (magnitudes) T_1, T_2, and T_3
- Use individual torque and RMS values + Performance curves provided by the manufacturer.
- Check heat generation + natural frequency of the drive.

Dynamics and Control Measures

- Rule of Thumb

$$\omega_n \leq \frac{1}{2}\omega_r \qquad \ldots (7.51)$$

ω_n : closed-loop natural frequency

ω_r : lowest structural resonant frequency

Manipulator Stiffness

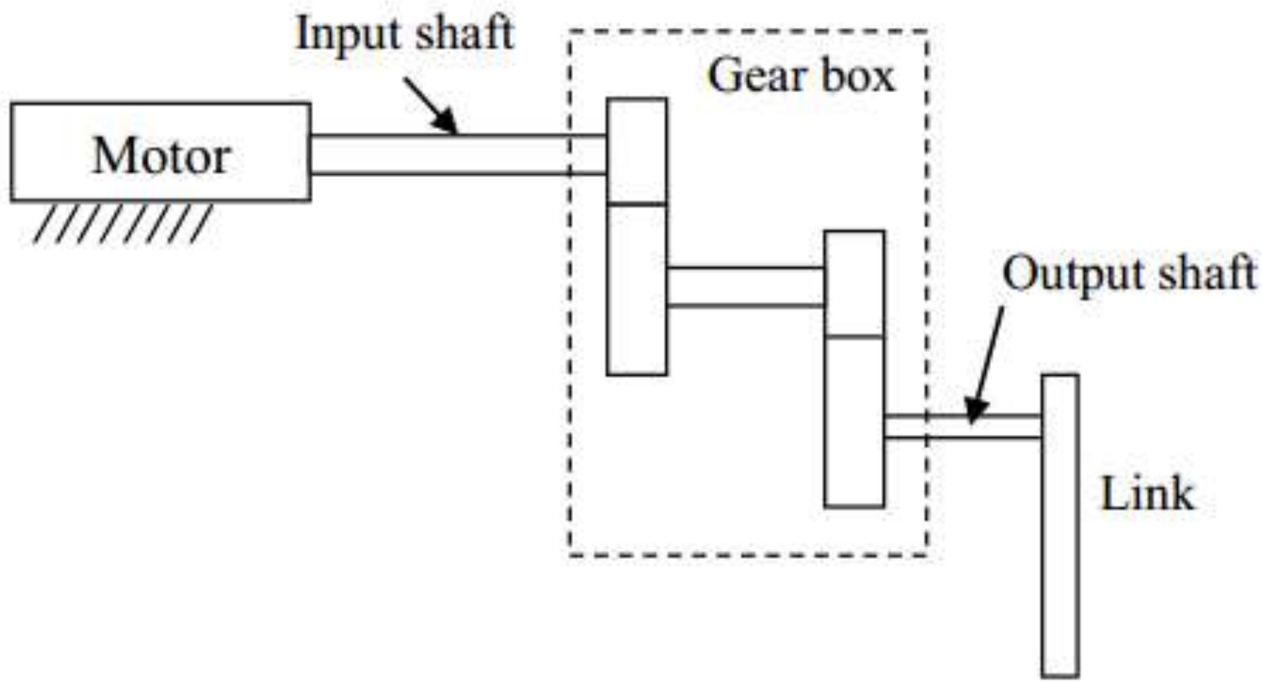


Figure 7.11 Shaft assembly of a link

$$\frac{1}{k_e} = \frac{1}{\eta^2 k_1} + \frac{1}{k_2} \quad \ldots (7.48)$$

$k_e \equiv$ equivalent stiffness

$\eta \equiv$ gear ratio

Link Material Selection

- Mild (low carbon) steel:

  S_y = 350 Mpa; S_u = 420 Mpa
- High alloyed steel

  S_y = 1750-1900 Mpa; S_u = 2000-2300 Mpa
- Aluminum
- S_y = 150-500 Mpa; S_u = 165-580 Mpa

Driver Selection

- Driver of a DC motor: A hardware unit which generates the necessary current to energize the windings of the motor
- Commercial motors come with matching drive systems

Summary

- Statics in robotics
- Manipulator design

Lecture 6 (SIT Sem. Rm.)

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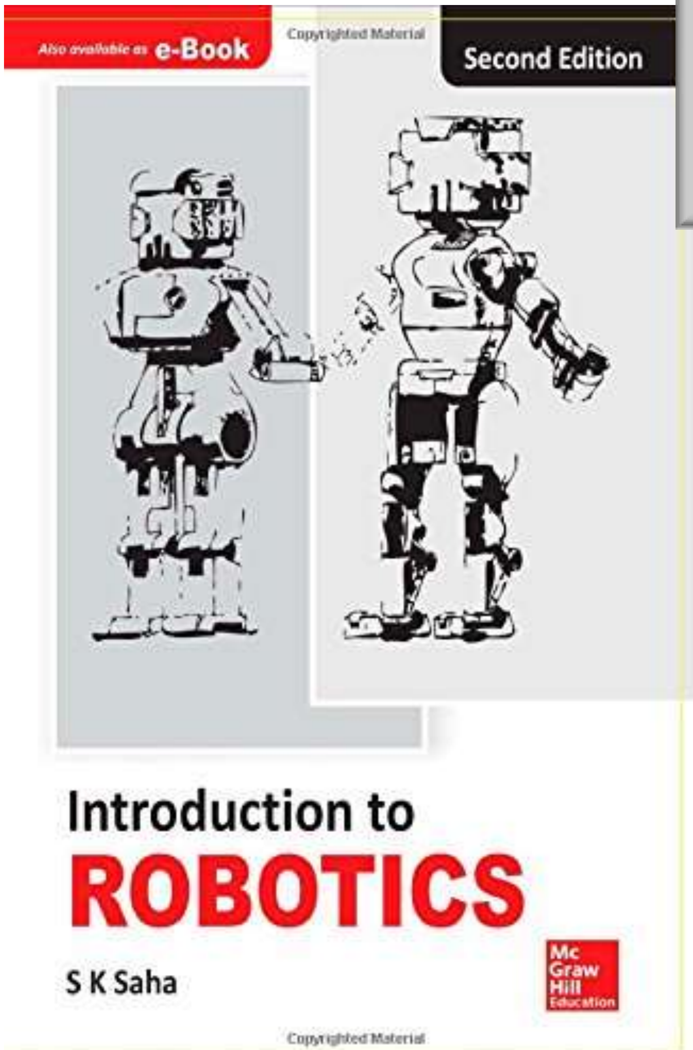


8

Dynamics

Outline

- Definition
- Euler-Lagrange Formulation
  - Generalized coordinates
  - Kinetic and potential energy
  - Equations of Motion

Euler-Lagrange Formulation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = \phi_i$$

L (Lagrangian) = $T - U$;

T: Kinetic energy; U: Potential energy;

q_i: Generalized coordinate;

ϕ_i : Generalized force.

Generalized Coordinates

- Coordinates that specify the configuration (position and orientation) → generalized coordinates

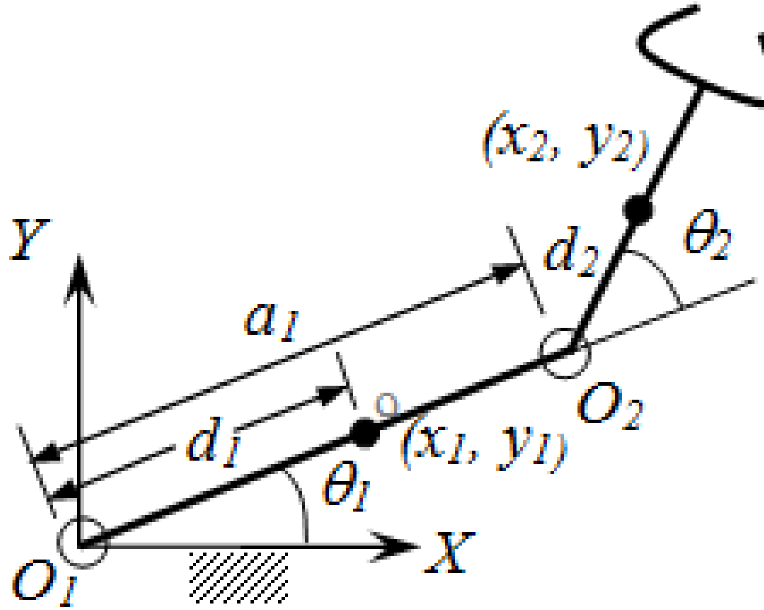


Figure 8.5 A two-link robot arm

Kinetic and Potential Energies

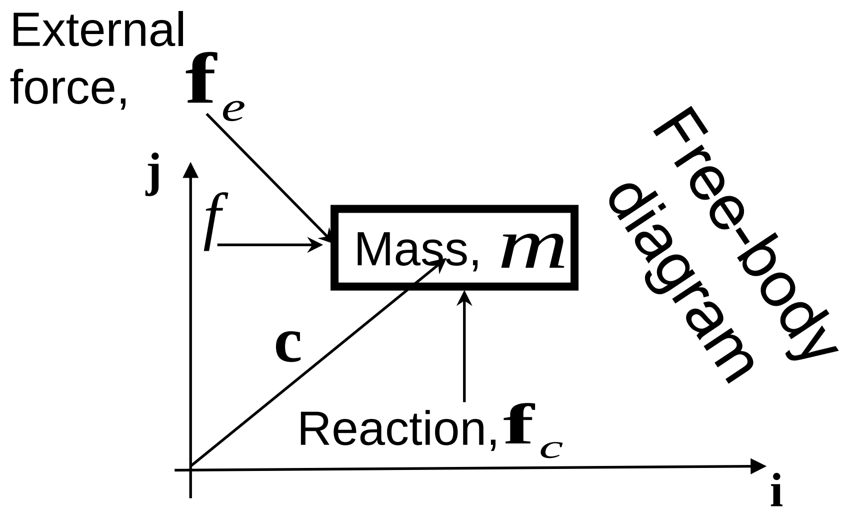
- Kinetic Energy

$$T = \sum_{i=1}^{n} T_i = \sum_{i=1}^{n} \frac{1}{2}\left(m_i \dot{\mathbf{c}}_i^T \dot{\mathbf{c}}_i + \boldsymbol{\omega}_i^T \mathbf{I}_i \boldsymbol{\omega}_i \right)$$

- Potential Energy

$$U = -\sum_{i=1}^{n} m_i \mathbf{c}_i^T \mathbf{g}$$

Euler-Lagrange Equation



Kinetic energy $$T = \frac{1}{2} m \dot{\mathbf{c}}^T \dot{\mathbf{c}}; U = 0$$

Velocity constraint: $\dot{\mathbf{c}} = \dot{x}\mathbf{i};$ $$L(= T - U) = \frac{1}{2} m \dot{x}^2$$

$$\frac{\partial L}{\partial \dot{x}} = m\dot{x}; \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = m\ddot{x}; \quad \frac{\partial L}{\partial x} = 0$$

Euler-Lagrange: $$m\ddot{x} = f$$

Example: One-DOF Arm (EL)

$T \equiv$ Please write!

$$U = mg\left(\frac{a}{2} - \frac{a}{2} c\theta\right)$$

$$L = T - U \equiv \quad\quad - mg\frac{a}{2}(1 - c\theta)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \quad\quad \frac{\partial L}{\partial \theta} = $$

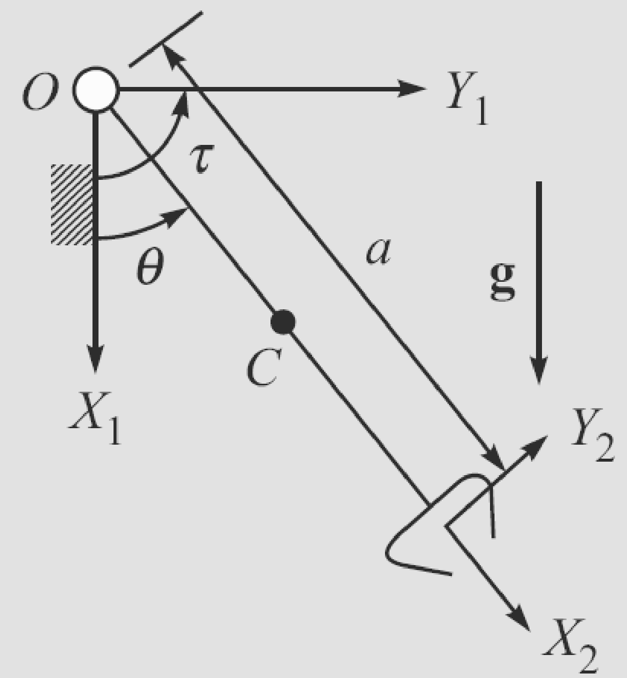


Fig. 8.7 One-link arm

Simulation of One-link Arm using MATLAB and MuPAD (contd…)

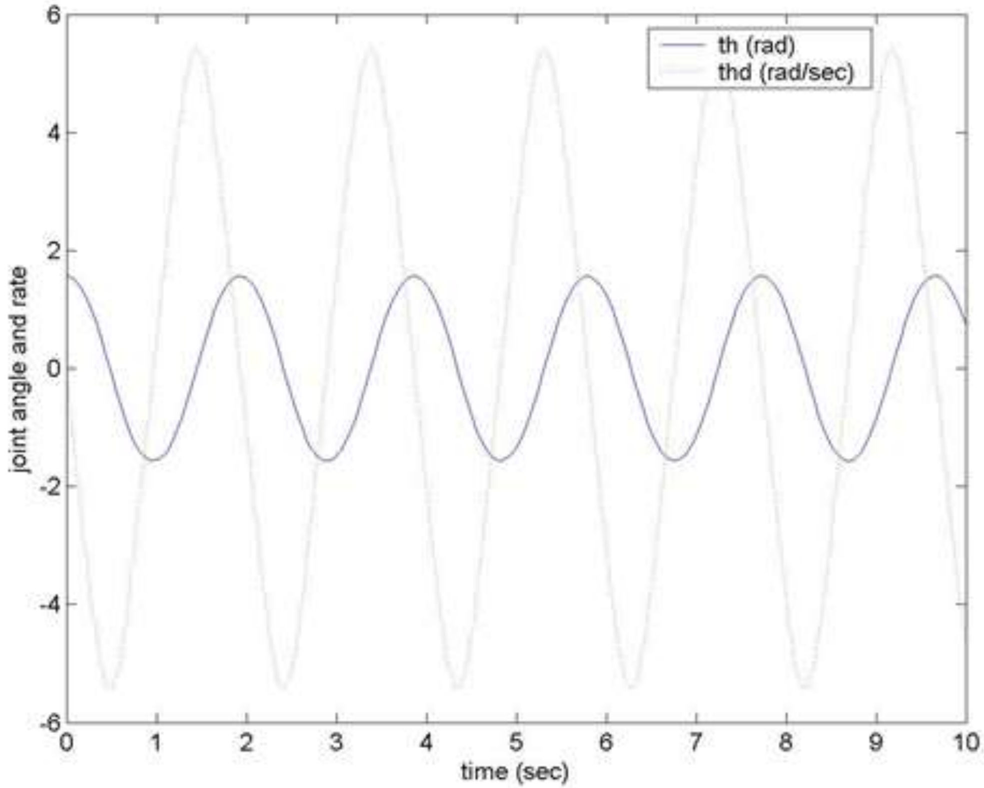


Figure 8.19 Simulation results of one-link arm under gravity

RoboAnalyzer

Simulation of One-link Arm using MATLAB

$$\ddot{\theta} = \frac{2}{ma^2}(\tau - \frac{1}{2} mga \sin\theta)$$

Hence, the state-space form is given by

$$\dot{y}_1 = y_2$$

$$\dot{y}_2 = \frac{2}{ma^2}(\tau - \frac{1}{2} mga \sin\theta)$$

```
%For one-link arm
function ydot=ch8fdyn1(t,y);
m = 1; a = 1; g = 9.81; tau=0;
iner = m*a*a/3; grav = m*g*a/2;
ydot=[y(2);(tau-grav*sin(y(1)))/iner];
```

(a) Program for state-space form

```
%For one link arm
tspan=[0 10]; y0=[pi/2; 0];
[t,y]=ode45('ch8fdyn1',tspan,y0)
```

(b) Program to integrate numerically

Figure 8.18 Simulation of one-link arm under gravity only

Mobile Robots

- Non-holonomic systems
  - Necessary and sufficient no. of variables defining a pose exceeds the number of actuators
- Holonomic
  - Necessary and sufficient no. of variables defining a pose is same as the no. independent actuators

Summary

- Euler-Lagrange equation was shown
  - Generalized coordinates, generalized forces were defined
- Demonstration with MATLAB and RoboAnalyzer
- Mobile Robot Dynamics

Lecture 7 (SIT Sem. Rm.)

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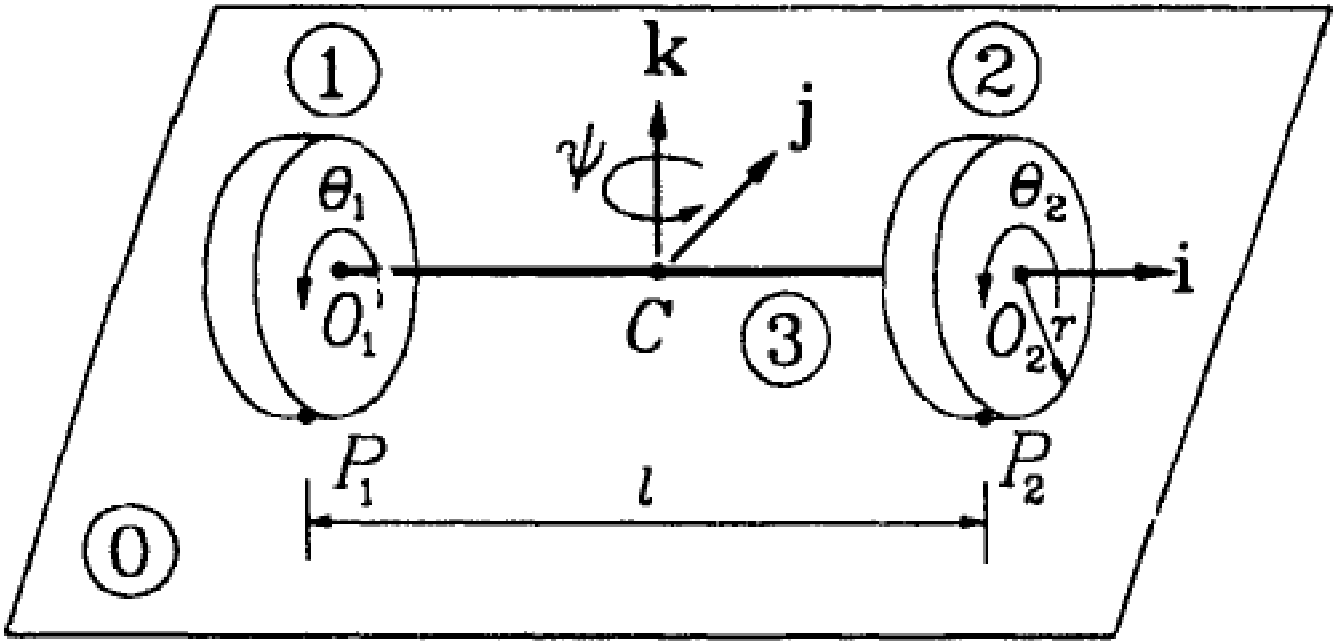
Mobile Robot Dynamics

[Ref: Dynamics and Design of Nonholonomic Robotic Mechanical Systems, Ph. D thesis, McGill Univ., Canada, 1991]

Two-wheeled System



Hand calculations on white board using Euler-Lagrange equation

Kinematic & Dynamic Models

$$\mathbf{t}_C = \mathbf{T}_C \dot{\boldsymbol{\theta}}_I$$

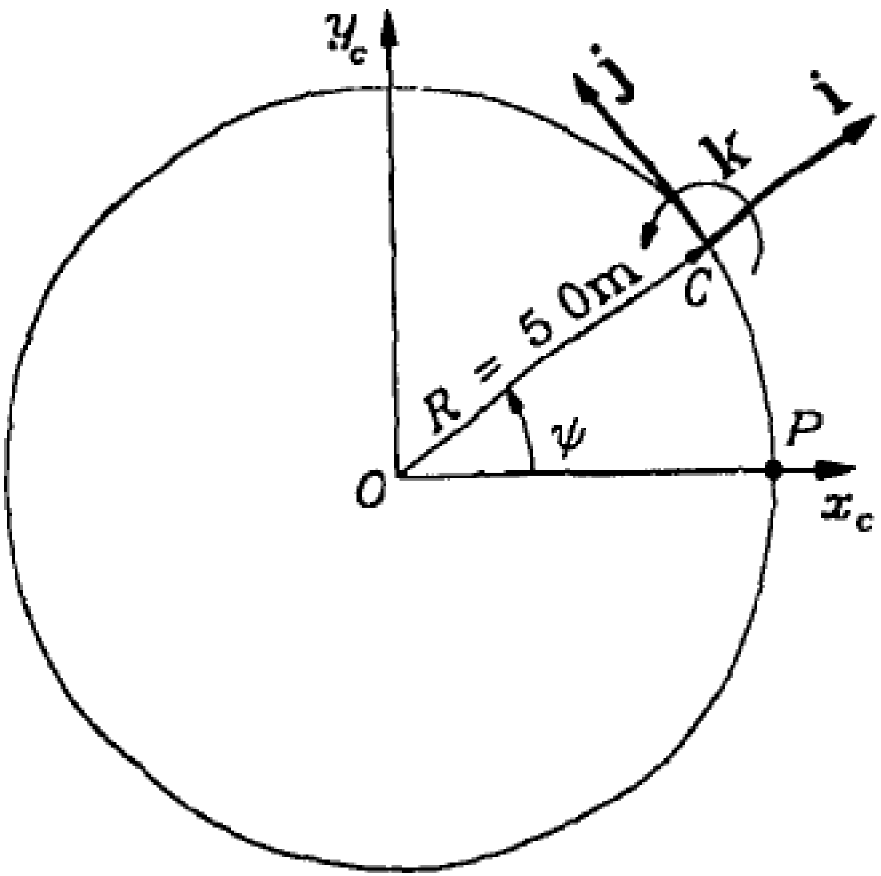
$$\mathbf{t}_C \equiv [\boldsymbol{\omega}^T, \ \mathbf{c}^T]^T \qquad \dot{\boldsymbol{\theta}}_I \equiv [\dot{\theta}_1, \ \dot{\theta}_2]^T \qquad \mathbf{T}_C = \frac{\eta}{2}\begin{bmatrix} 2\mathbf{k} & -2\mathbf{k} \\ -l\mathbf{j} & -l\mathbf{j} \end{bmatrix}$$

$$\eta = r/l$$

$$\mathbf{I}\ddot{\boldsymbol{\theta}}_I = \boldsymbol{\tau}$$

$$\mathbf{I} = \frac{mr^2}{2}\begin{bmatrix} 3+\eta^2 & -\eta^2 \\ -\eta^2 & 3+\eta^2 \end{bmatrix}. \qquad \boldsymbol{\tau} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

Circular Path

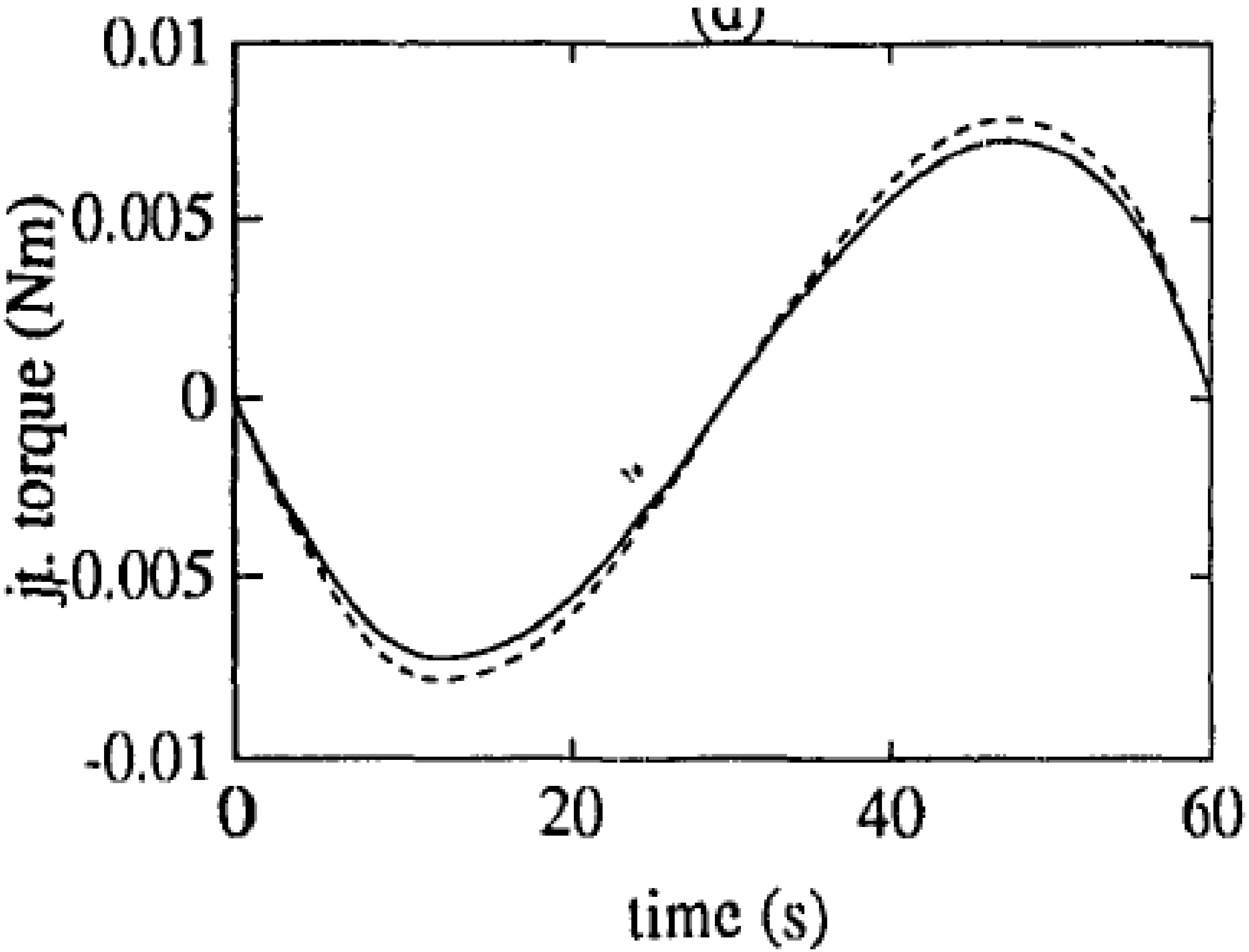


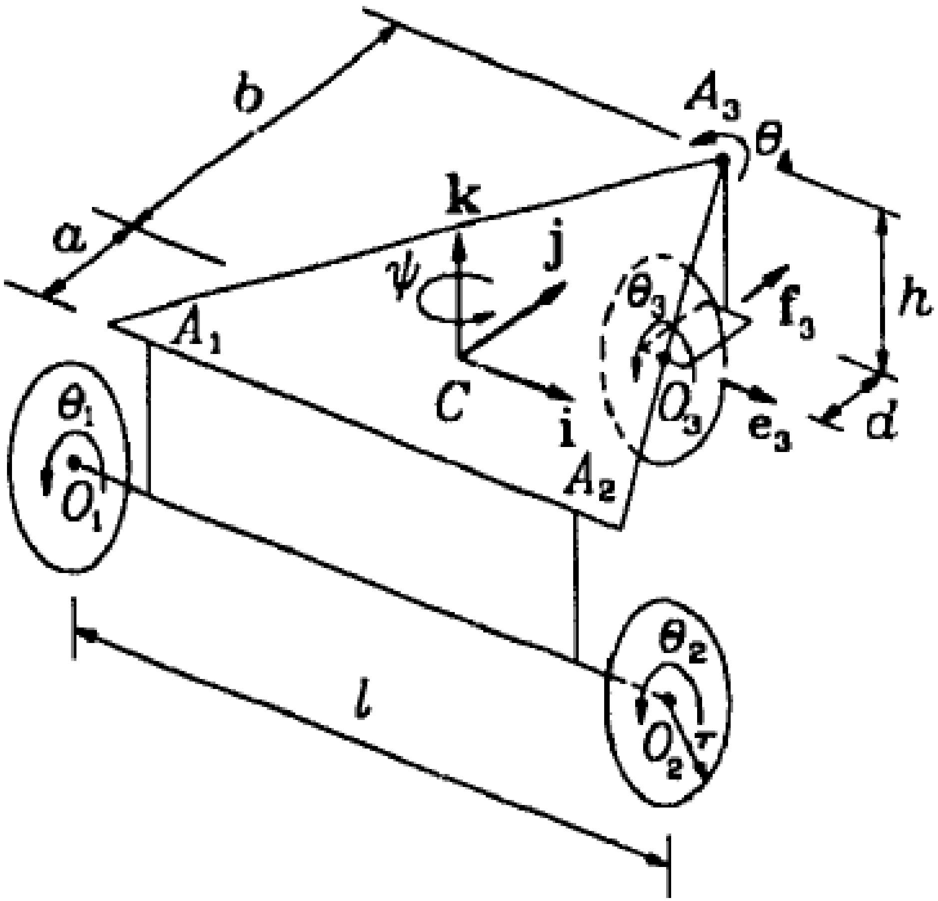
$r = 0.05$ m,

$l = 0.4$ m,

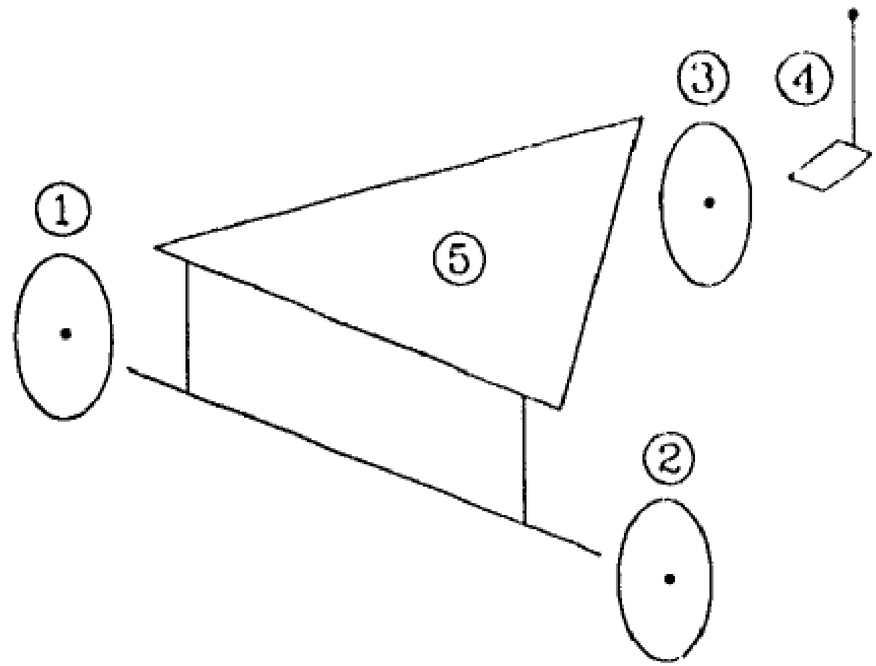
$m = 2.0$ kg

Joint Torques (- 1; .. 2)



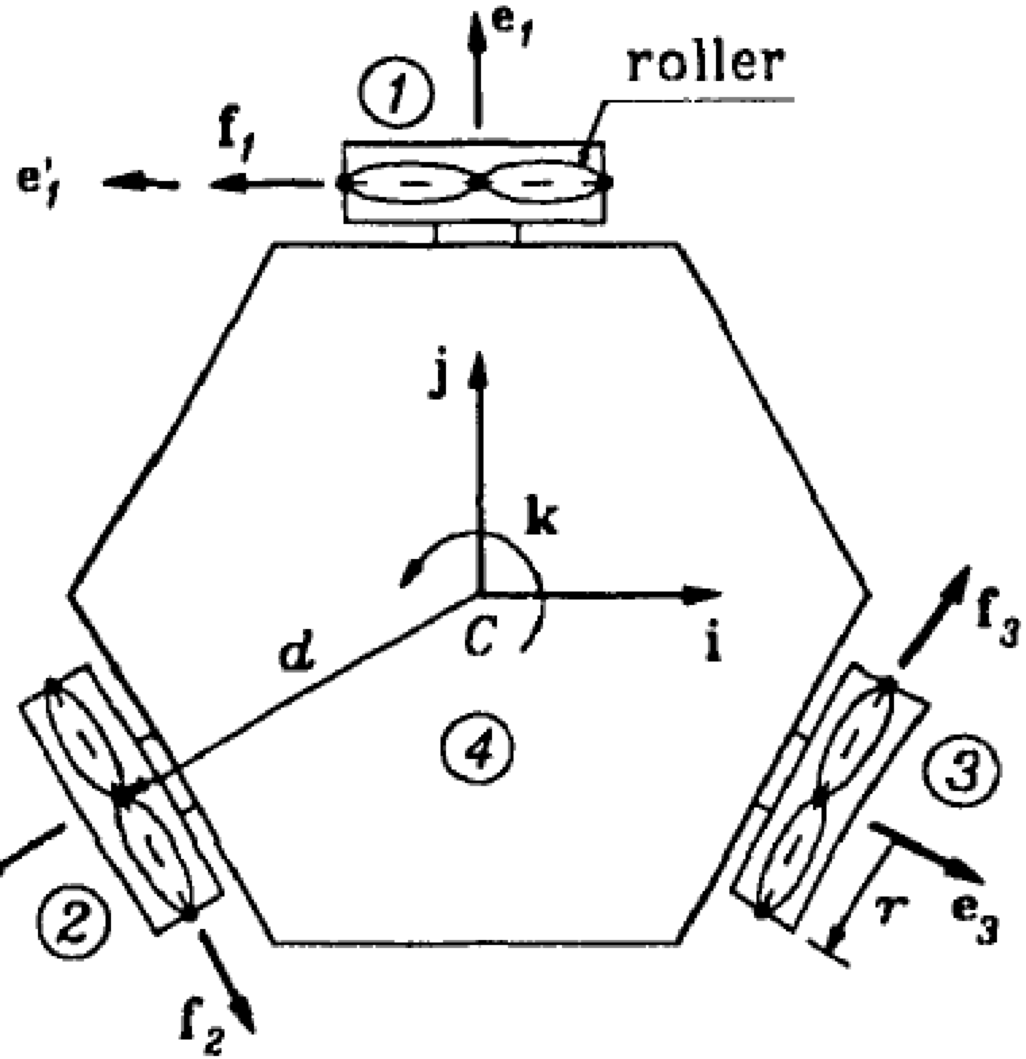


b
a
A3
θ4
k
j
ψ
θ3
f3
h
A1
C
i
O3
e3
d
θ1
O1
A2
θ2
O2
l

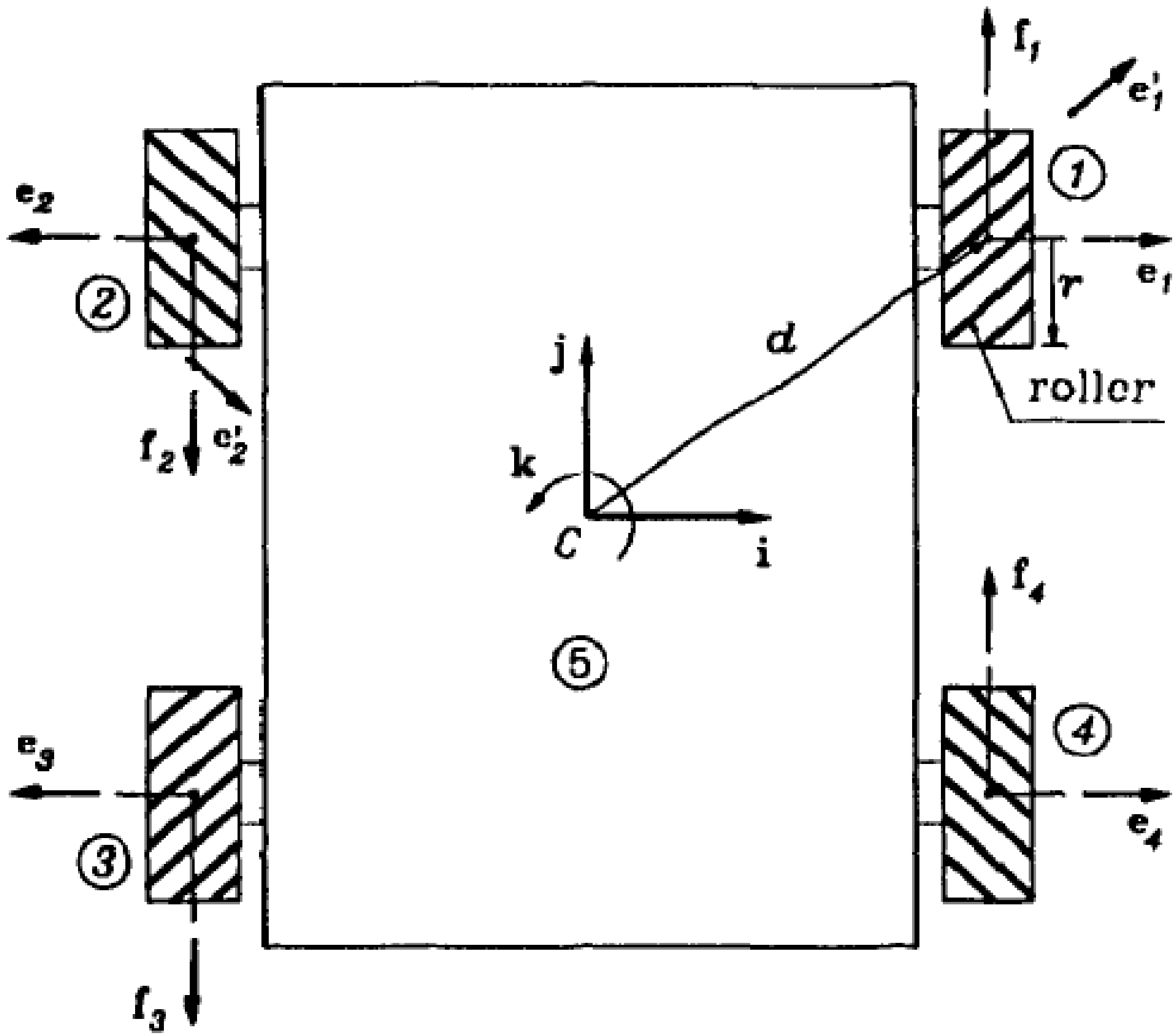


1
2
3
4
5

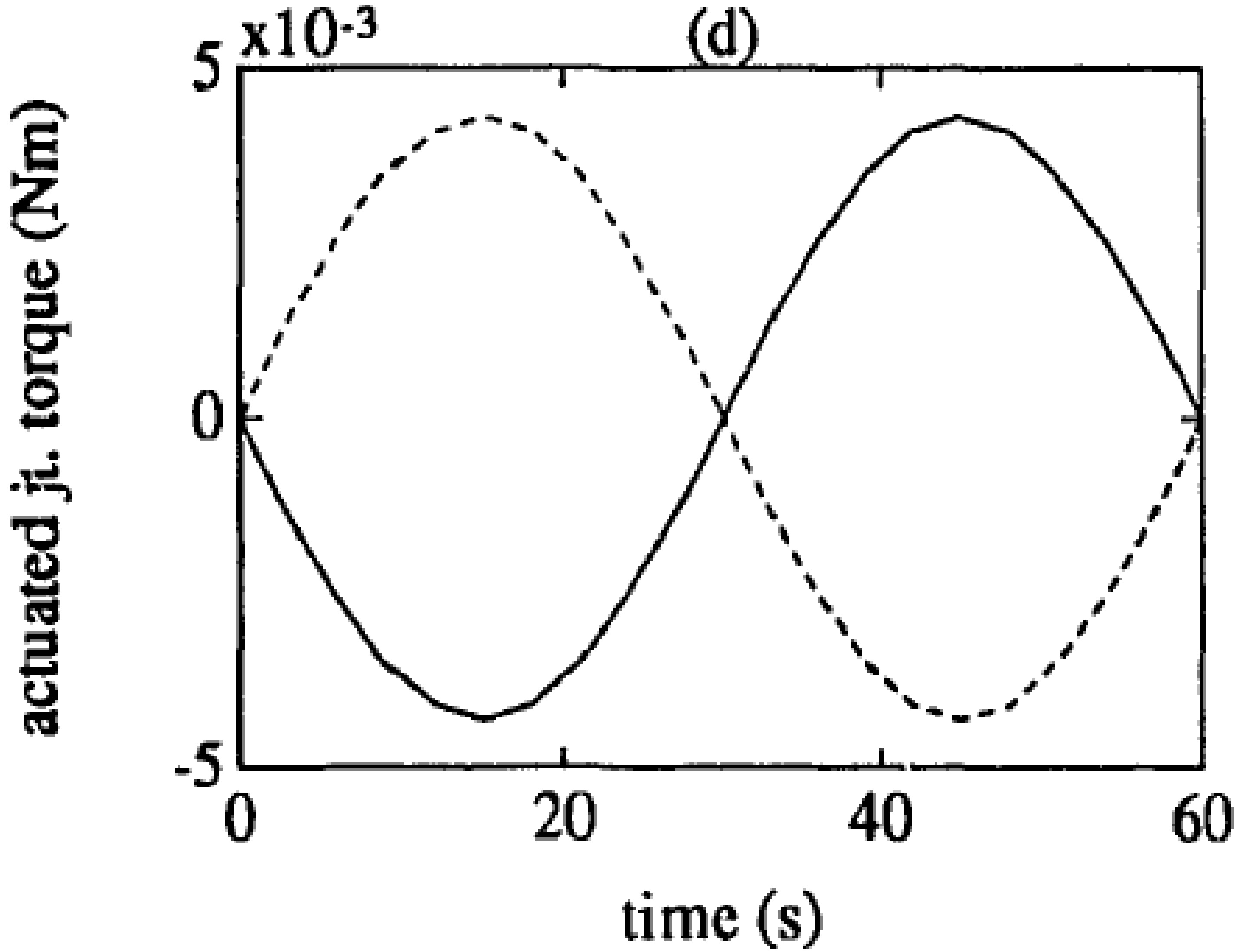
Three-DOF 3-Wheeled Mobile Robot



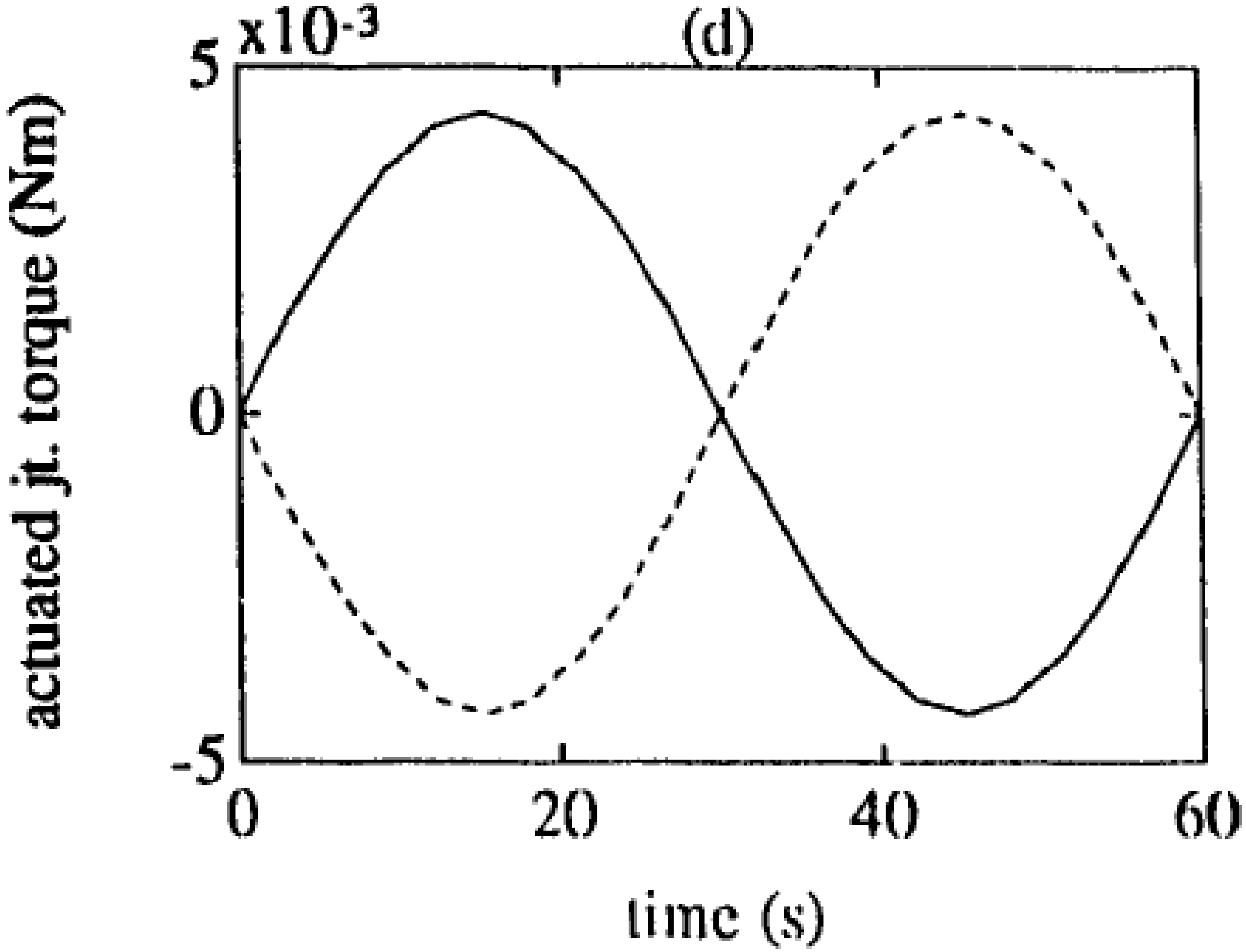
Three-DOF 4-Wheeled



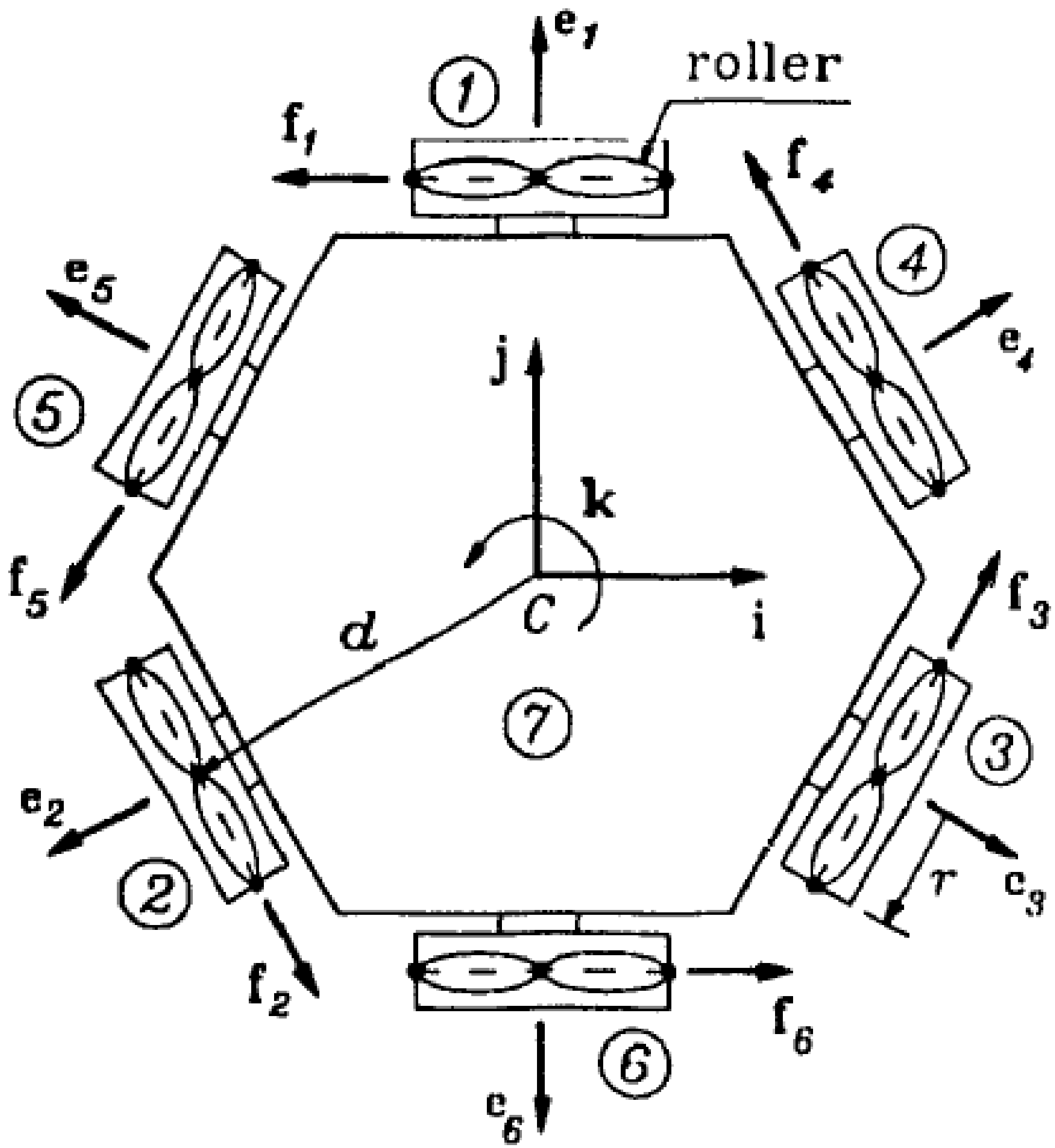
Joint Torques@**j** (- 1&4; .. 2&3)



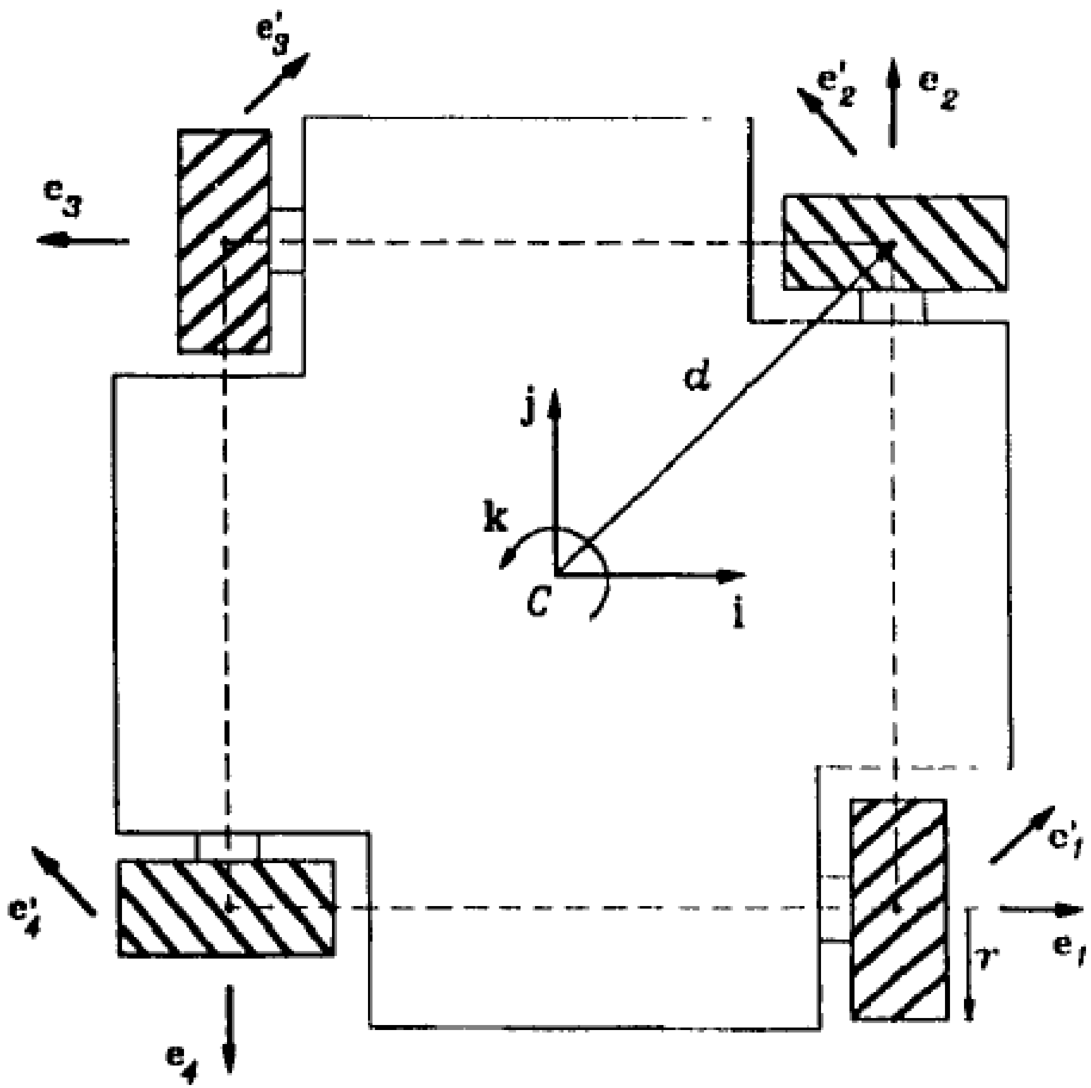
Joint Torques@i (-1&2; .. 3&4



Three-DOF 6-Wheeled



Isotropic 3-DOF 4-Wheeled



Summary

- Mobile robots are nonholonomic systems
- Dynamic model for a 2-wheeled system
- Several mobile robots with omnidirectional wheels are shown
- Isotropic design was emphasized

THANK YOU

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http://sksaha.com