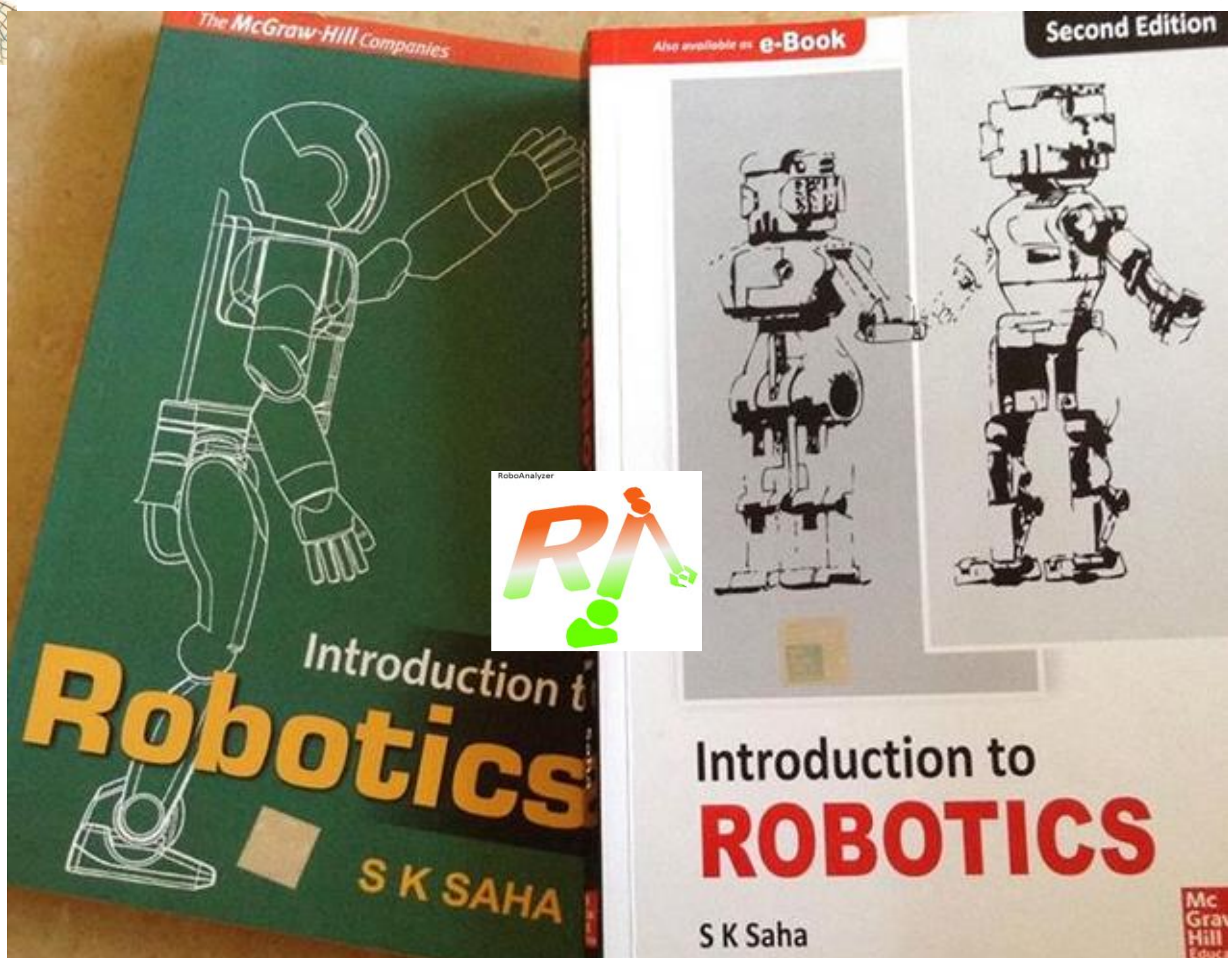
Lecture 05

Forward and Inverse Kinematics



VLFM Course Module on Robotics

February 18, 2020

Announcement

- Lectures 1-4 are available in

  http://sksaha.com/courses

Review of Lecture 4

- **Coordinate Transformation**
  - **Relation between two coordinate frames**
  - **Homogeneous Transformation Matrix (HTM)**
- **Forward kinematics**
- **Use of RoboAnalyzer**

Outline

- Examples for Forward Kinematics
- Inverse kinematics
  - Multiple solutions

Kinematics

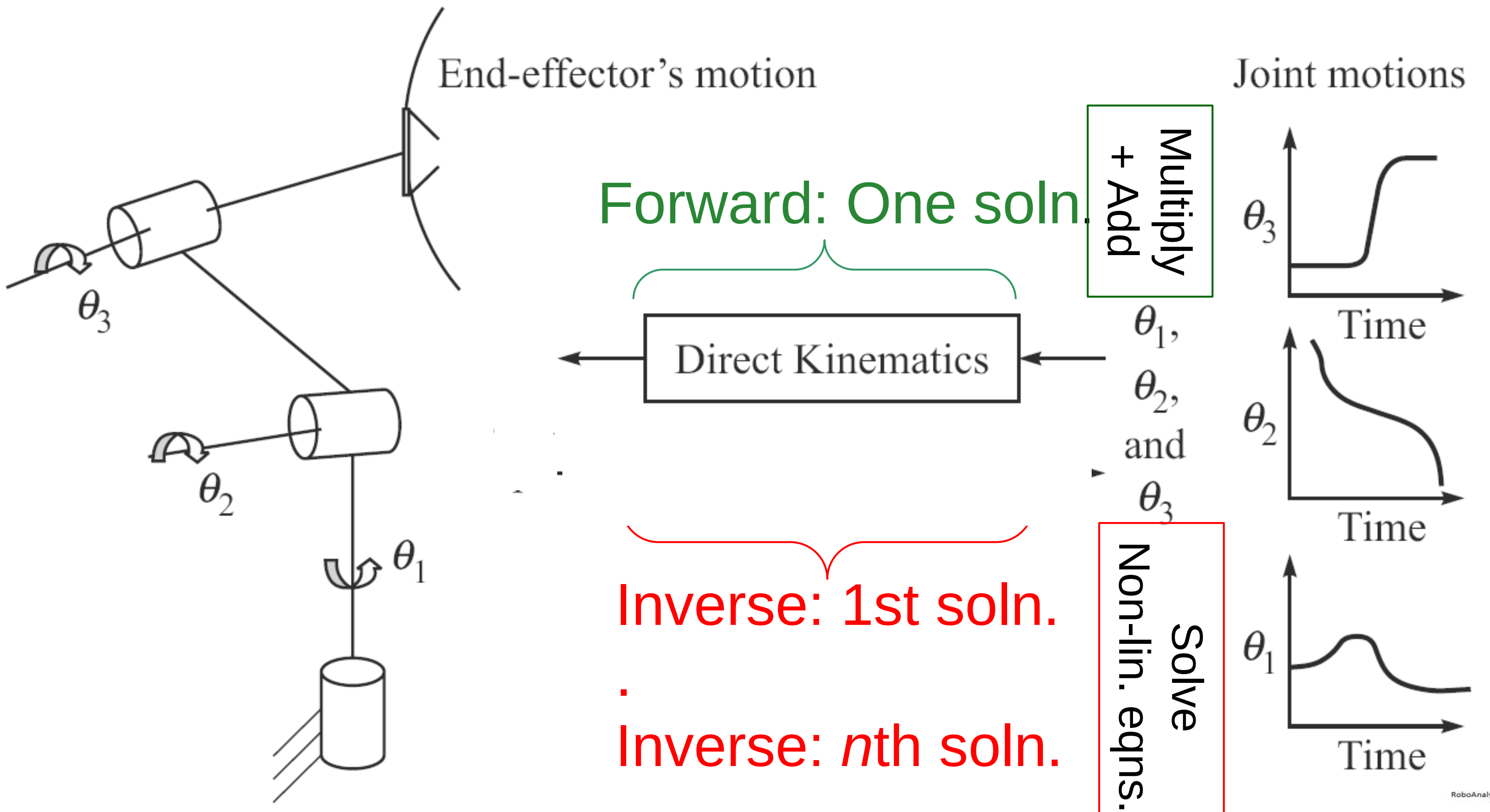


Fig. 6.1 Forward and inverse kinematics

- Forward kinematics relation

$$\mathbf{T} = \mathbf{T}_1 \, \mathbf{T}_2 \ldots \mathbf{T}_n$$

$\mathbf{T}_i =$

DH Parameters and Matrices for 2-link Planar Arm

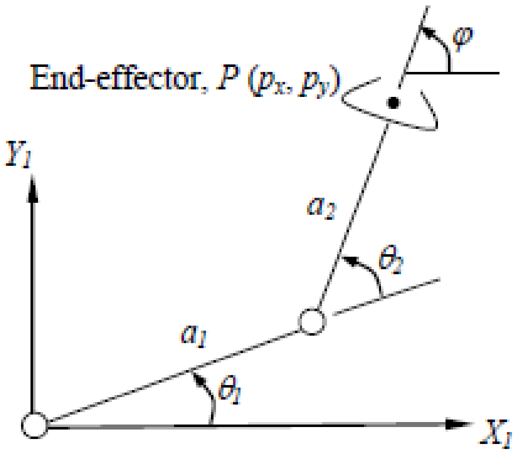


Figure 6.2 Kinematics of a two-link planar arm

Link	b_i	θ_i	a_i	α_i
1	0	θ_1 (JV)	a_1	0
2	0	θ_2 (JV)	a_2	0

For i = 1,2

$$\mathbf{T}_i \equiv \begin{bmatrix} c_i & -s_i & 0 & a_i c_i \\ s_i & c_i & 0 & a_i s_i \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\mathbf{T} = \mathbf{T}_1 \mathbf{T}_2$

$$\mathbf{T} \equiv \begin{bmatrix} c_{12} & -s_{12} & 0 & a_1 c_1 + a_2 c_{12} \\ s_{12} & c_{12} & 0 & a_1 s_1 + a_2 s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ..(6.5)$$

Examples

RoboAnalyzer

Inverse Kinematics: Two-link Arm

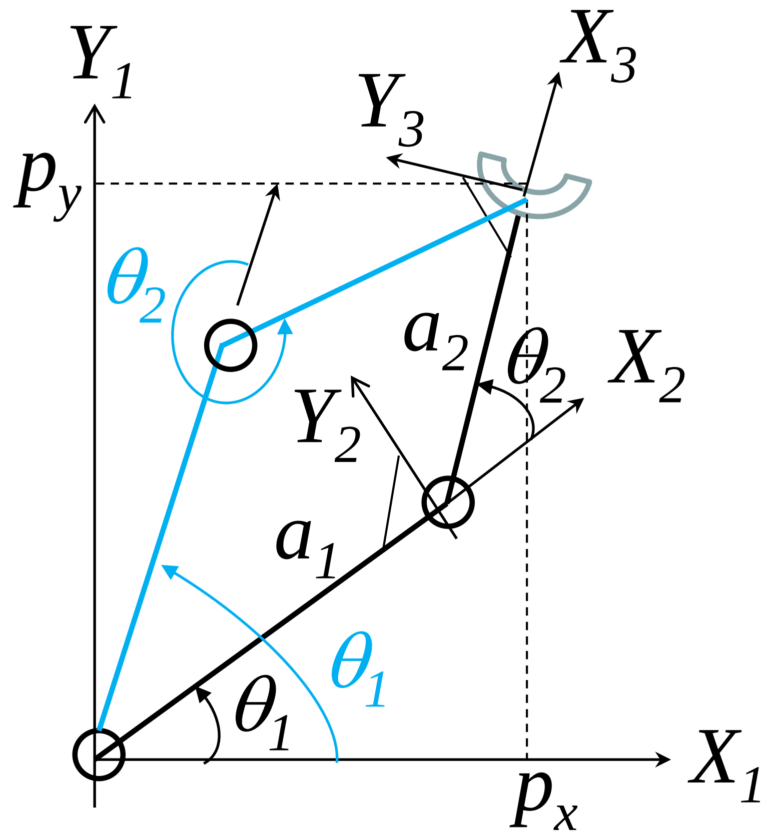
$$p_x = a_1 c_1 + a_2 c_{12}$$

$$p_y = a_1 s_1 + a_2 s_{12}$$

$$c_2 = \frac{p_x^2 + p_y^2 - a_1^2 - a_2^2}{2 a_1 a_2}$$

$$s_2 = \pm\sqrt{1 - c_2^2}$$

Verify it!



RoboAnalyzer

$$\theta_2 = \text{atan2}\,(s_2, c_2)$$

$$s_1 = \frac{(a_1 + a_2 c_2) p_y - a_2 s_2 p_x}{\Delta}$$

$$\Delta \equiv a_1^2 + a_2^2 + 2 a_1 a_2 c_2 = p_x^2 + p_y^2$$

$$c_1 = \frac{(a_1 + a_2 c_2) p_x + a_2 s_2 p_y}{\Delta}$$

$$\theta_1 = \text{atan2}\,(s_1, c_1)$$

Numerical Example

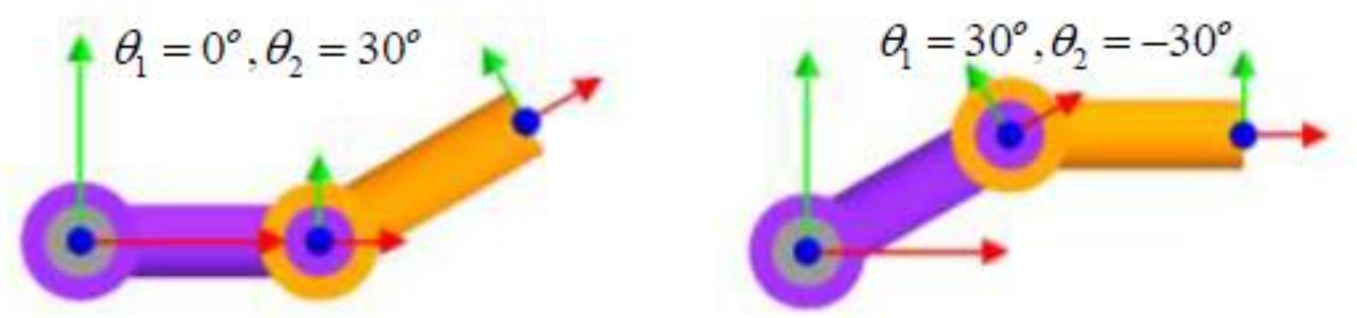
- Inverse Kinematics of Two-link Planar Arm (Example 6.14)

Input:

$$\mathbf{T} \equiv \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & 1+\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The inverse kinematics solution for the 2-link planar arm using RoboAnalyzer is given below:

Verify!



(a) Elbow down (b) Elbow up

Figure 6.9 Inverse kinematics solution of two-link planar arm

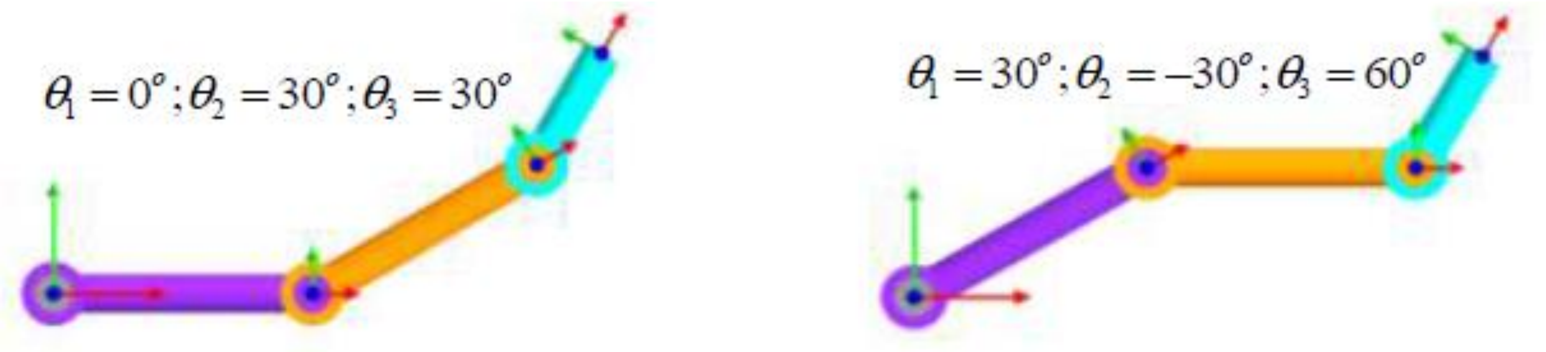
Numerical Example

- Inverse Kinematics of the Three-link Planar Arm (Example 6.15)

Input:

$$\mathbf{T} \equiv \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 & \sqrt{3}+\frac{5}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & \frac{\sqrt{3}}{2}+1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The inverse kinematics solution for the 3-link planar arm using RoboAnalyzer is given below:



(a) Elbow down (b) Elbow up

Figure 6.10 Inverse kinematics solution of three-link planar arm

Three-DOF Articulated Arm

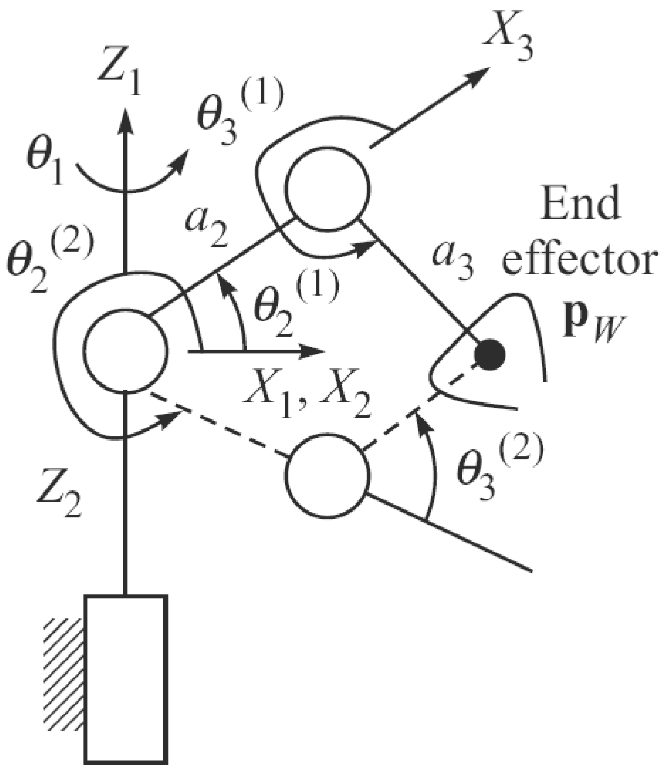
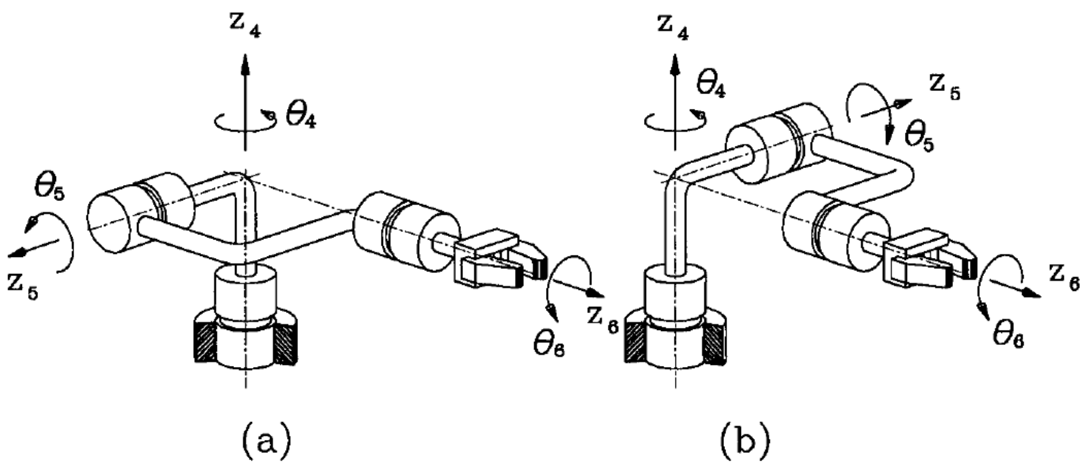


Fig. 6.7 Two admissible solutions

4 solutions!

Three-DOF Wrist



2 solutions!

KUKA has 4x2=8 solutions

Conclusions

- Examples of Forward and Inverse Kinematics
- Unique solution for FK
- Multiple solutions for IK
- Illustrations with RoboAnalyzer software

THANK YOU

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