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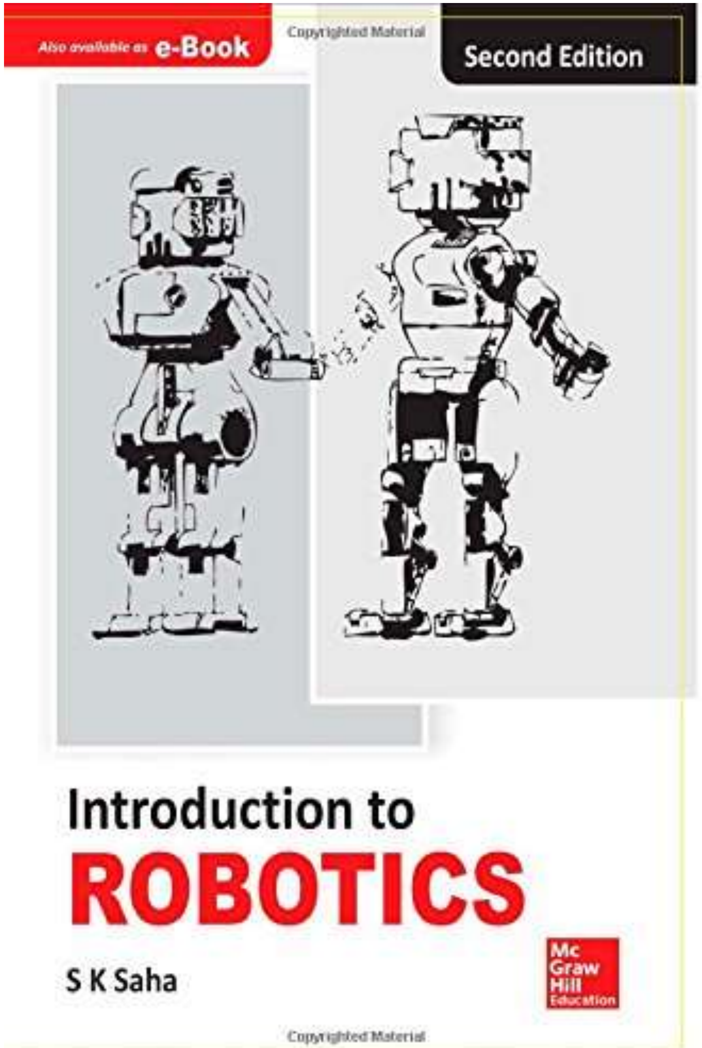

Lecture 5

Velocity Analysis (Ch. 6): Jacobian

by

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Course Project: Mobile Platform

- Build a two-wheeled driven mobile robot platform
- Perform kinematic analysis, i.e., given a path of the platform what are the wheel motions required [SK Saha]
- Attach a camera for some application (navigation?) [S Dutta Roy]
- Apply some form of control [IN Kar]
- Implement in some hardware! [Kolin Paul]

Velocity Analysis: Jacobian

Jacobian maps joint rates into end-effector's velocities. It depends on the manipulator configuration.

$$\text{twist of end-effector}: \mathbf{t}_e \equiv \begin{bmatrix} \boldsymbol{\omega}_e \\ \mathbf{v}_e \end{bmatrix}; \text{ Joint rates}: \dot{\boldsymbol{\theta}} = \begin{bmatrix} \dot{\theta}_1 \\ \vdots \\ \dot{\theta}_n \end{bmatrix}$$

$$\mathbf{t}_\mathbf{e} = \mathbf{J}\dot{\boldsymbol{\theta}} \qquad \text{where } \mathbf{J} = \begin{bmatrix} \mathbf{j}_1 & \mathbf{j}_2 & \cdots & \mathbf{j}_n \end{bmatrix} \text{ and}$$

$$\mathbf{J} = \begin{bmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \cdots & \mathbf{e}_\mathbf{n} \\ \mathbf{e}_1 \times \mathbf{a}_{1e} & \mathbf{e}_2 \times \mathbf{a}_{2e} & \cdots & \mathbf{e}_\mathrm{n} \times \mathbf{a}_{ne} \end{bmatrix} \qquad \text{. . (6.86)}$$

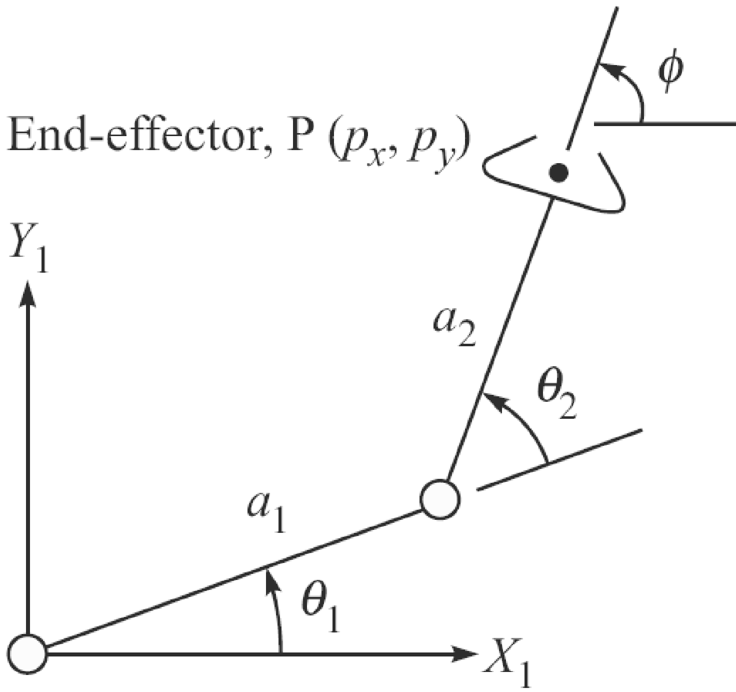
$$\mathbf{j}_\mathrm{i} \equiv \begin{bmatrix} \mathbf{e}_\mathrm{i} \\ \mathbf{e}_\mathrm{i} \times \mathbf{a}_\mathrm{ie} \end{bmatrix}, \text{ if Joint } i \text{ is revolute} \quad \mathbf{j}_i \equiv \begin{bmatrix} \mathbf{0} \\ \mathbf{e}_i \times \mathbf{a}_{ie} \end{bmatrix}, \text{ if Joint } i \text{ is prismatic}$$

Jacobian of a 2-link Planar Arm

$$\mathbf{J} = \begin{bmatrix} \mathbf{e}_1 \times \mathbf{a}_{1e} & \mathbf{e}_2 \times \mathbf{a}_{2e} \end{bmatrix}$$

where $\mathbf{e}_1 \equiv \mathbf{e}_2 \equiv [0 \quad 0 \quad 1]^T$

$$\mathbf{a}_{1e} \equiv \mathbf{a}_1 + \mathbf{a}_2$$
$$\equiv [a_1c_1 + a_2c_{12} \quad a_1s_1 + a_2s_{12} \quad 0]^T$$
$$\mathbf{a}_{2e} \equiv \mathbf{a}_2$$
$$\equiv [a_2c_{12} \quad a_2s_{12} \quad 0]^T$$



Hence, $\mathbf{J} = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \end{bmatrix}$

Check from position expression also!

Example: Singularity of 2-link RR Arm

$$\mathbf{J} \equiv \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \end{bmatrix} \qquad \theta_2 = 0 \text{ or } \pi$$

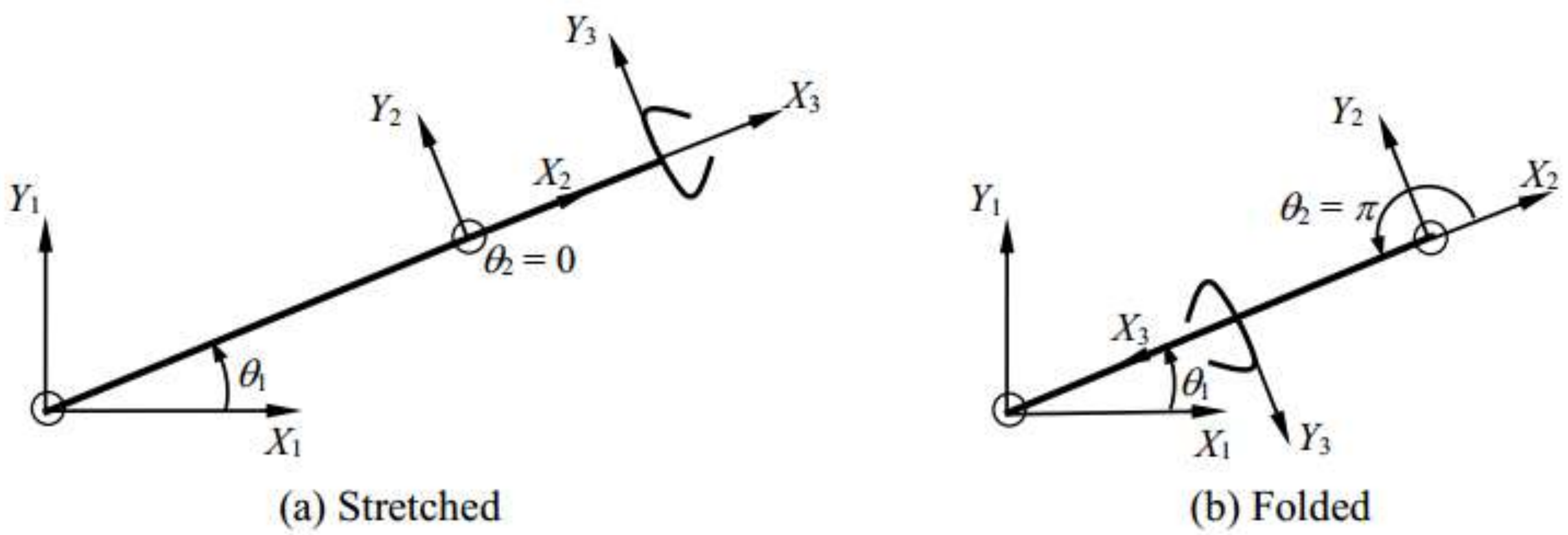


Figure 7.9 Singular configurations of a two-link planar arm

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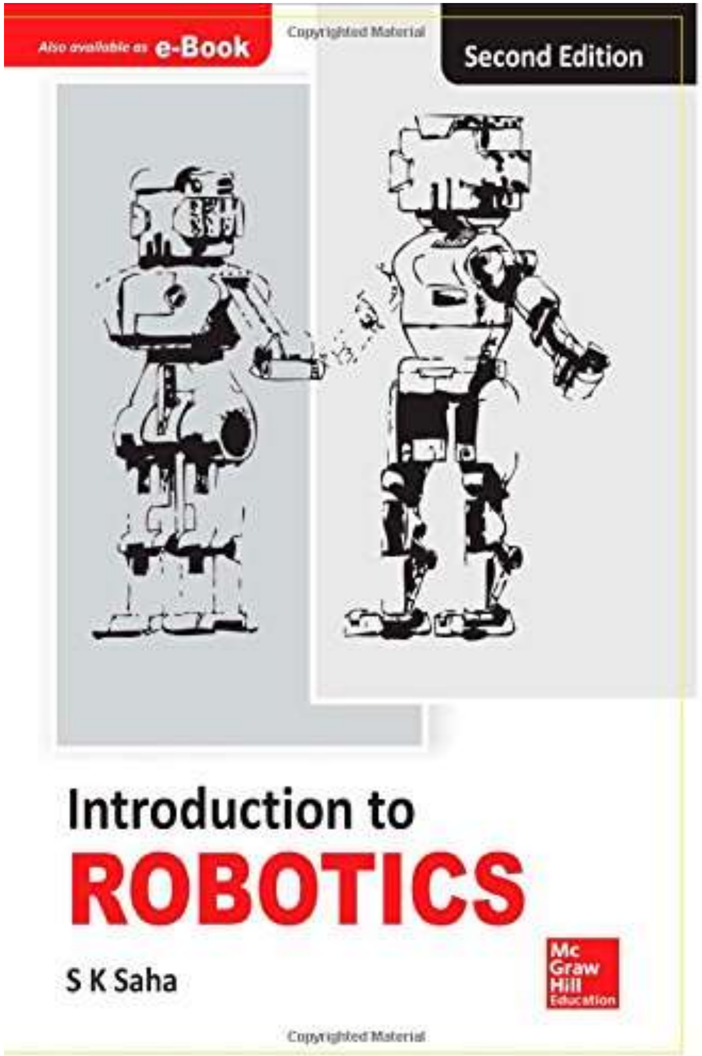


Jacobian in Statics (Ch. 7)

Principle of Virtual Work

$$\mathbf{w}_e^T \delta \mathbf{x} = \boldsymbol{\tau}^T \delta \boldsymbol{\theta} \quad \text{... (7.28)}$$

- Relation between two virtual displacements (Can be derived from velocity expression)

$$\delta \mathbf{x} = \mathbf{J} \delta \boldsymbol{\theta} \quad \text{... (7.29)}$$

$$\mathbf{w}_e^T \mathbf{J} \delta \boldsymbol{\theta} = \boldsymbol{\tau}^T \delta \boldsymbol{\theta} \Rightarrow \mathbf{w}_e^T \mathbf{J} = \boldsymbol{\tau}^T \quad \text{... (7.31)}$$

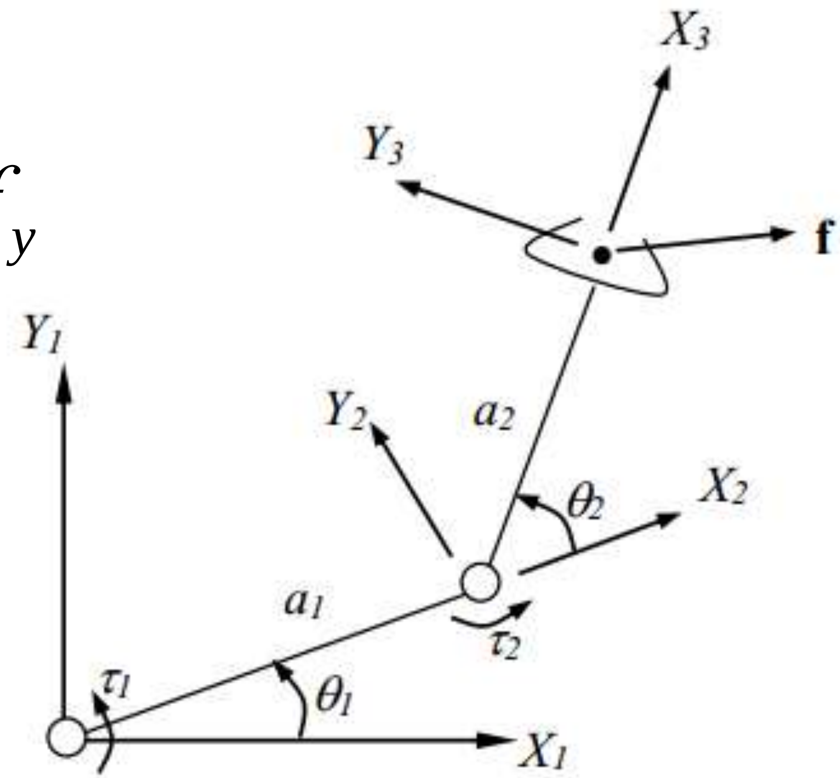
$$\boxed{\boldsymbol{\tau} = \mathbf{J}^T \mathbf{w}_e} \quad \text{... (7.32)}$$

Example: 2-link RR Planar Arm

$$\tau_1 = [\mathbf{e}_1]_1^T [\mathbf{n}_{01}]_1$$
$$= a_1 f_x s\theta_2 + (a_2 + a_1 c\theta_2) f_y$$

$$\tau_2 = [\mathbf{e}_2]_2^T [\mathbf{n}_{12}]_2 = a_2 f_y$$

$$\boldsymbol{\tau} = \mathbf{J}^{\mathrm{T}} \mathbf{f}$$



$$\boldsymbol{\tau} \equiv \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \quad \mathbf{J}^T \equiv \begin{bmatrix} a_1 s\theta_2 & a_1 c\theta_2 + a_2 & 0 \\ 0 & a_2 & 0 \end{bmatrix} \quad \mathbf{f} \equiv \begin{bmatrix} f_x \\ f_y \\ 0 \end{bmatrix}$$

Two Jacobian Matrices

- From Statics

$$\mathbf{J} \equiv \begin{bmatrix} a_1 s\theta_2 & 0 \\ a_1 c\theta_2 + a_2 & a_2 \\ 0 & 0 \end{bmatrix}$$

- From Kinematics

$$\mathbf{J} \equiv \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \end{bmatrix}$$

Jacobian from Statics in Frame 1

$$[\mathbf{J}]_1 \equiv \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 \\ s\theta_2 & c\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_1 s\theta_2 & 0 \\ a_1 c\theta_2 + a_2 & a_2 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -a_1 s\theta_1 - a_2 s\theta_{12} & -a_2 s\theta_{12} \\ a_1 c\theta_1 + a_2 c\theta_{12} & a_2 c\theta_{12} \\ 0 & 0 \end{bmatrix} \quad \ldots (7.34)$$

- Without the last row, it is the same as the one from kinematics ← Should be!

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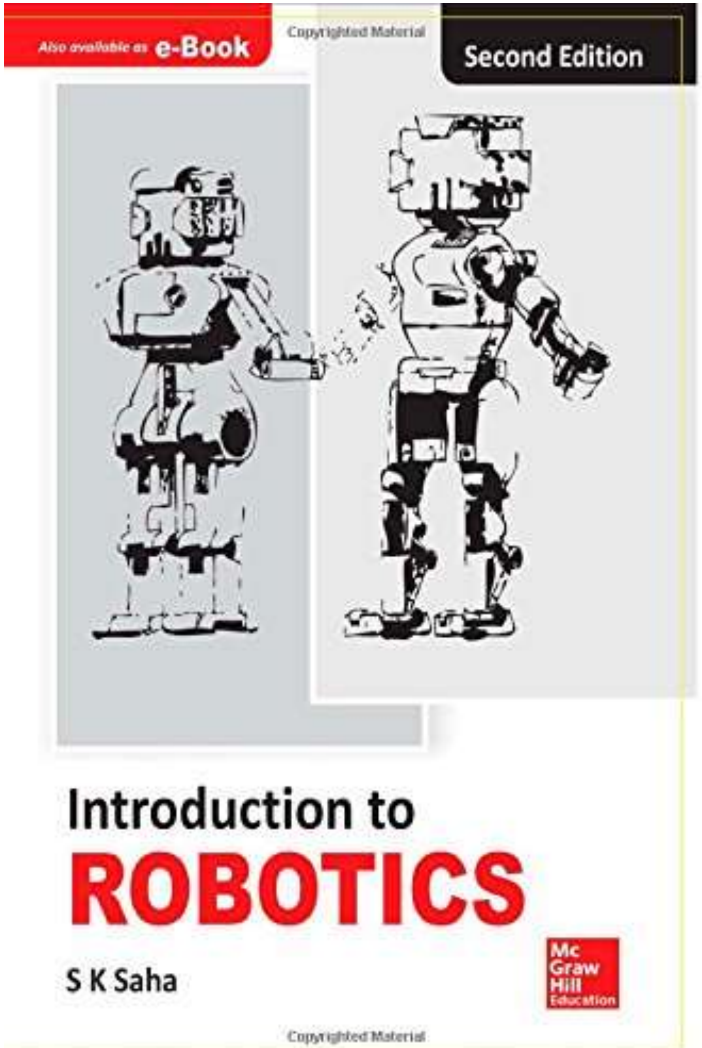


Dynamics (Ch. 8)

Euler-Lagrange Formulation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = \phi_i$$

L (Lagrangian) = $T - U$;

T: Kinetic energy; U: Potential energy;

q_i: Generalized coordinate;

ϕ_i : Generalized force.

Generalized Coordinates

- Coordinates that specify the configuration (position and orientation) → generalized coordinates

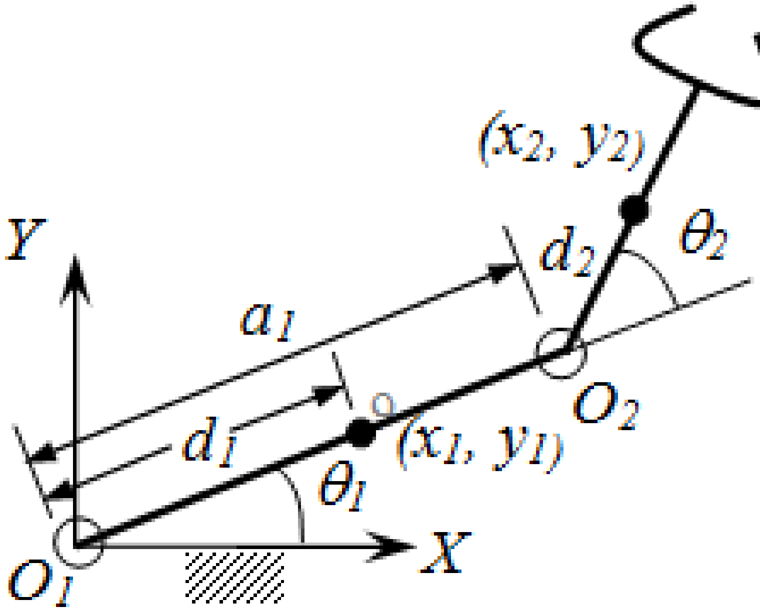


Figure 8.5 A two-link robot arm

Kinetic and Potential Energies

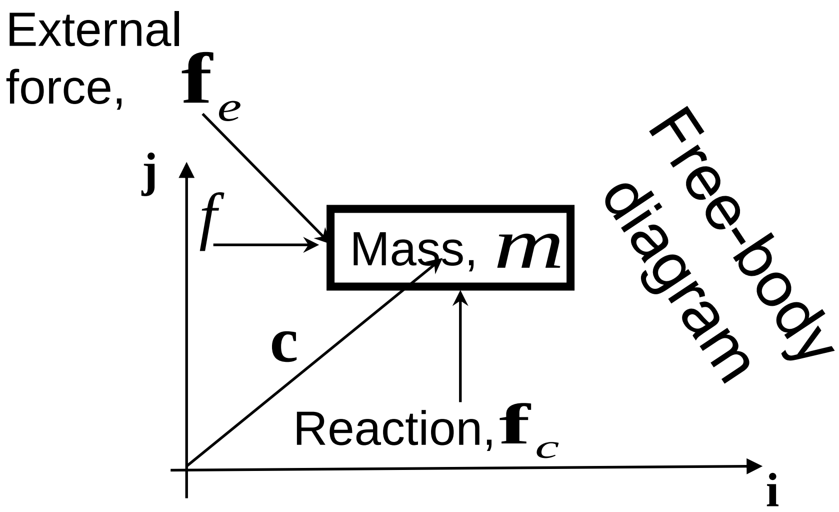
- Kinetic Energy

$$T = \sum_{i=1}^{n} T_i = \sum_{i=1}^{n} \frac{1}{2}\left(m_i \dot{\mathbf{c}}_i^T \dot{\mathbf{c}}_i + \boldsymbol{\omega}_i^T \mathbf{I}_i \boldsymbol{\omega}_i \right)$$

- Potential Energy

$$U = -\sum_{i=1}^{n} m_i \mathbf{c}_i^T \mathbf{g}$$

Euler-Lagrange Equation



Kinetic energy $T = \frac{1}{2} m \dot{\mathbf{c}}^T \dot{\mathbf{c}};\ U = 0$

Velocity constraint: $\dot{\mathbf{c}} = \dot{x}\mathbf{i};$ $L(= T - U) = \frac{1}{2} m \dot{x}^2$

$$\frac{\partial L}{\partial \dot{x}} = m\dot{x}; \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = m\ddot{x}; \qquad \frac{\partial L}{\partial x} = 0$$

Euler-Lagrange: $\boxed{m\ddot{x} = f}$

Example: One-DOF Arm (EL)

$$T \equiv \frac{1}{2} m(\frac{a}{2}\dot{\theta})^2 + \frac{1}{2}\frac{ma^2}{12}\dot{\theta}^2;$$

$$U = mg(\frac{a}{2} - \frac{a}{2}c\theta)$$

$$L = T - U \equiv \frac{ma^2}{6}\dot{\theta}^2 - mg\frac{a}{2}(1 - c\theta)$$

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{\theta}}) = \frac{1}{3} ma^2 \ddot{\theta}; \frac{\partial L}{\partial \theta} = -\frac{1}{2} mgas\theta$$

$$\frac{1}{3} ma^2 \ddot{\theta} + \frac{1}{2} mgas\theta = \tau$$

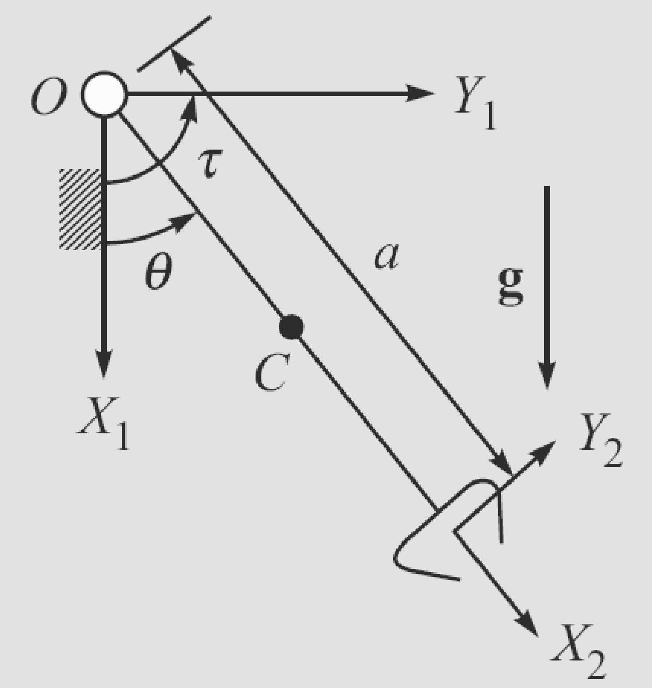


Fig. 8.7 One-link arm

Simulation of One-link Arm

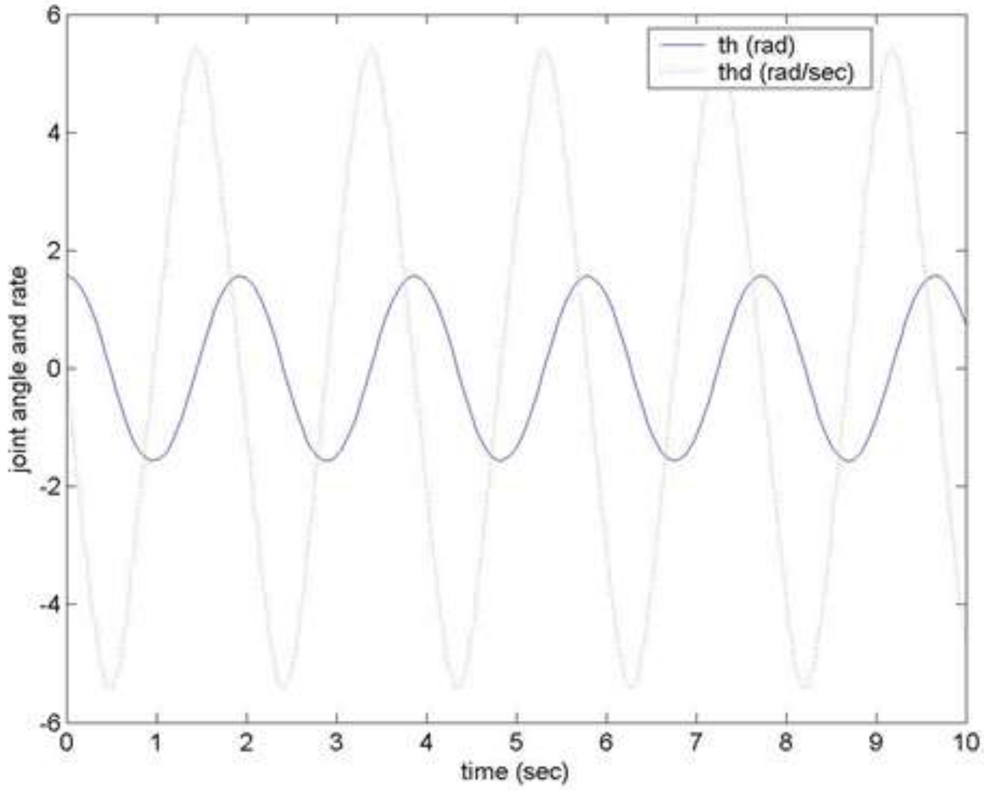


Figure 8.19 Simulation results of one-link arm under gravity

RoboAnalyzer

Simulation of One-link Arm using MATLAB

$$\ddot{\theta} = \frac{2}{ma^2}(\tau - \frac{1}{2} mga \sin\theta)$$

Hence, the state-space form is given by

$$\dot{y}_1 = y_2$$

$$\dot{y}_2 = \frac{2}{ma^2}(\tau - \frac{1}{2} mga \sin\theta)$$

```
%For one-link arm
function ydot=ch8fdyn1(t,y);
m = 1; a = 1; g = 9.81; tau=0;
iner = m*a*a/3; grav = m*g*a/2;
ydot=[y(2);(tau-grav*sin(y(1)))/iner];
```

(a) Program for state-space form

```
%For one link arm
tspan=[0 10]; y0=[pi/2; 0];
[t,y]=ode45('ch8fdyn1',tspan,y0)
```

(b) Program to integrate numerically

Figure 8.18 Simulation of one-link arm under gravity only

THANK YOU

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