Identification of Denavit-Hartenberg Parameters of an Industrial Robot

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***Abstract*— Kinematic identification of a serial robot has been an active field of research as the need for improving the accuracy of a robot is increasing with time. Denavit-Hartenberg (DH) parameters of a serial robot, which are typically used to represent its architecture, are usually provided by its manufacturer. At times these parameters are not the same and hence they need to be identified. An analytical method proposed elsewhere was used here for identification of an industrial robot by noting the values of the point on the end-effector due to rotation of each joint, locking all other joints, were found out using singular value decomposition. The DH parameters of the robot determined using the proposed methodology, matched satisfactorily with the robot specifications. Also, the bounding volume for the joint range infers that a smaller measurement volume relative to the robot workspace is required thus facilitating the use of measurement devices which have smaller range of measurement.**

Keywords*— *Kinematic identification; DH parameters; Dual vector algebra; Singular Value Decomposition; 3D circle fit

I. Introduction

Industrial robots extensively use serial architecture to perform tasks such as pick and place, painting, arc welding, assembly of components etc. As these operations require complex motion to be followed, exact knowledge of the kinematic parameters are very essential for absolute positioning, accuracy and repeatability. Now-a-days tasks needed to be performed by the robots are defined using analytical or off-line programming tools also. So there is a requirement for accurate correspondence between the robot model and the robot in action.

The relation between the joint and task spaces of the robot is done with the help of geometric model. The most widely used notation is Denavit-Hartenberg (DH) [1] and is generally provided by the robot manufacturers in the form of robot specifications. To identify these specifications of the robot in action whose parameters may differ due to manufacturing and assembly errors of components, wear and tear, etc., measurement devices with a very high accuracy in long range is required. Method of calibration and kinematic identification using different measuring methods is presented in [2]. Position measurement devices with high accuracy of taking readings at a large distance are expensive. So, there exists a need to identify the accurate model of the robot using a method which can utilize the smaller workspace measurement inputs.

The most important task in reducing the positioning error is to eliminate the difference between the real robot geometry and the geometric representation stored in the controller. Some of the methods for geometric model representation are Hayati parameters [3] where a parameter was added to take into account the effect of parallel or near-parallel consecutive joint axes for avoiding numerical instabilities during estimation. S-model method was introduced in [4], for describing and characterizing kinematics of robotic manipulator. The widely used methods to calibrate the inaccurate robot are presented in the survey papers [5-6]. The error due to geometric factors accounted for 95% of the positioning error [7]. To find the geometric parameters accurately, the most critical step is the data acquisition of the robot pose which is also a time consuming process. The process of identification requires use of instruments such as theodolite [8], 3D coordinate measurement device [9], ultrasonic range sensors as measurement device was presented in [4].

An analytical method using Vector Algebra to extract the DH parameters was proposed in [10]. The method provided motivation to develop a novel methodology using Dual Vector Algebra in [11], which has compact and elegant representation. An extension work for identification without base calibration is presented in [12]. Using the method in [11], the autonomous identification method is proposed here which is only based on the joint sensor readings obtained through the robot sensory interface of an industrial robot, say KUKA KR5. The position of the end-effector caused by the movement of only one joint keeping the others locked were noted, and the process was repeated by progressing towards the inward joints starting from the last one. In [13], only the first three links measurements were taken into account for a 180° rotation of each joint. In this paper, all six links measurements were taken into account. The method is useful in identifying the DH parameters of the robot within a small workspace, by giving a limited angular motion to each joint. As a result, one can use measurement devices with lesser measuring volume, thus increasing the accuracy and reducing the cost of the device.

This paper is divided into three sections. In section II, the general definition, identification algorithm using the singular value decomposition and least square method are introduced. Then in Section III, data acquisition of position of a point on end-effector of an industrial robot from an industrial robot is shown with the identification and results. Concluding remarks are presented in Section IV.

II. FORMULATION

This section explains in brief the formulation used behind the identification of the DH parameters.

A. Geometric Description using DH Parameters

The most widely used notation for the geometric modeling of robots is Denavit-Hartenberg (DH) notation [14]. The travel from the base frame to the end-effector frame is achieved by moving across two consecutive frames placed at the joints. The set of four parameters relates the transformation between Frame *i* to Frame *i*+1 by b_i, θ_i, a_i and α_i, as shown in Fig 1.

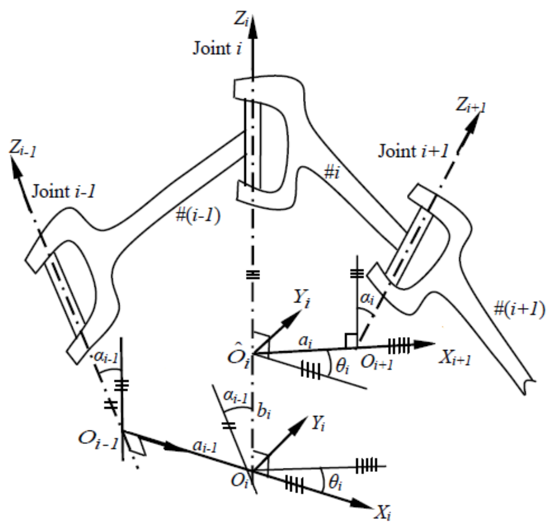


Fig. 1. DH parameters and Frames attached

TABLE I. SYMBOLIC NOTATION USED TO DESCRIBE THE DH PARAMETERS WITH ITS DEFINITION.

Symbol	Name	Description
b_i	Joint offset	$X_i \xrightarrow[@Z_i]{\perp,\text{distance}} X_{i+1}$
θ_i	Joint Angle	$X_i \xrightarrow[@Z_i]{\text{rotation, ccw}} X_{i+1}$
a_i	Link Length	$Z_i \xrightarrow[@X_{i+1}]{\perp,\text{distance}} Z_{i+1}$
α_i	Twist Angle	$Z_i \xrightarrow[@X_{i+1}]{\text{rotation, ccw}} Z_{i+1}$

*In the table read symbol $\longrightarrow$ as "to", $\perp$ as "perpendicular", @ as "along" and *ccw* as "counter clockwise".

B. Extraction of DH parameters

Denavit-Hartenberg (DH) [1] parameters of an industrial robot are usually provided by its manufacturer either in the form of specifications or engineering drawings. An analytical method was proposed in [11] to determine the DH parameters from the CAD model of a robot. The methodology was developed as an addin/plugin inside Autodesk Inventor CAD software. As an input, it required the joint axes direction and the coordinates of the center of the circle described by joint motion, measured in a frame attached to the base of the robot. Using the Application Programming Interface (API) of the software, the joint axes, i.e., a point on the axis (C_i) and its direction (Z_i), were determined as illustrated in Fig. 2. These axes were represented as Dual Vectors and the relationship between two consecutive joint axes, i.e., if they are parallel, intersecting or skewed, was determined using Dual Vector Algebra and the DH parameters were determined using line geometry. The methodology is used here to determine the DH parameters of a real robot, namely, KUKA KR5.

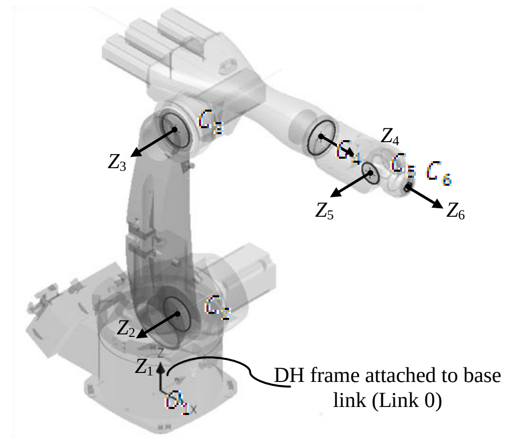


Fig. 2. Joint axes of CAD model of KUKA KR5 robot

C. Identification of circular feature

This section explains the method for finding the joint axis by knowing the normal and center of the circle formed by the set of 3D data points in the space. The set of *m* data points were obtained experimentally from the method explained in section III (A), denoted by say, (x_i, y_i, z_i for i=1, 2… m). Fig. 3 depicts the spread of data points in the 3D space with respect to a measurement frame. The method to find the equation of fitted circle and its normal to the plane shown in Fig. 3 is described here.

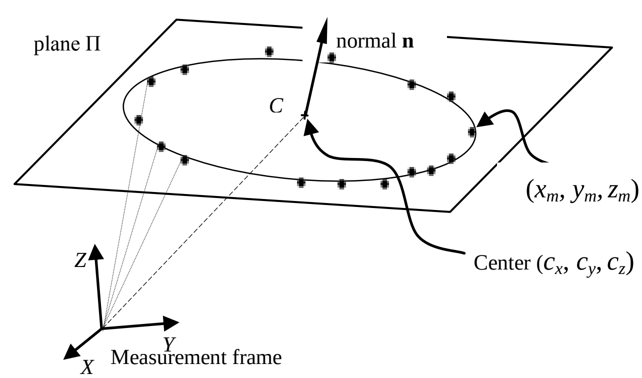


Fig. 3. Circle in 3D space with its center and normal.

The centroid of the set of all the data points measured with respect to the frame is found as using

$$\bar{\mathbf{x}} = [\bar{x}, \bar{y}, \bar{z}]^T \equiv \frac{1}{m}\left[\sum x_i, \sum y_i, \sum z_i\right] \quad (1)$$

The data points are then put inside a matrix denoted as **M,** and then it is transformed with respect to the centroid as denoted by matrix **N**, i.e.,

$$\mathbf{M} \equiv \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ \vdots & \vdots & \vdots \\ x_m & y_m & z_m \end{bmatrix} ; \ \mathbf{N} \equiv \begin{bmatrix} x_1 - \bar{x} & y_1 - \bar{y} & z_1 - \bar{z} \\ x_2 - \bar{x} & y_2 - \bar{y} & z_2 - \bar{z} \\ \vdots & \vdots & \vdots \\ x_m - \bar{x} & y_m - \bar{y} & z_m - \bar{z} \end{bmatrix} \quad (2)$$

Note that the Singular Value Decomposition (SVD) of matrix N gives two orthogonal matrices **U** and **V** and one diagonal matrix containing positive singular values of **S**, i.e., σ_1, σ_2, and σ_3. As shown in Fig. 4 the magnitude of singular values represents the length of axes. The SVD of the matrix **N** is given by

$$\text{SVD}(\mathbf{N}) = \mathbf{U}_{m\times m}\mathbf{S}_{m\times 3}\mathbf{V}^{\mathrm{T}}_{3\times 3} \quad (3)$$

The first two columns of $\mathbf{V} \equiv (\mathbf{v_1}\ \mathbf{v_2}\ \mathbf{v_3})$, namely $\mathbf{v}_1$ and $\mathbf{v}_2$, give an orthonormal basis for the plane Π shown in Fig. 4.

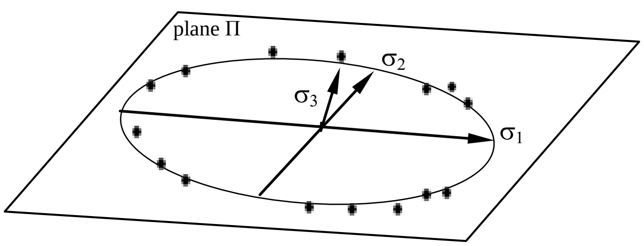


Fig. 4. Spread of 3D data points with the singular values along mutually perpendicular directions

The third column $\mathbf{v}_3$ corresponds to the lowest singular value σ_3, which gives normal **n.** The set of 3D data points were then mapped to the 2D plane formed by direction vectors $\mathbf{v}_1$ and $\mathbf{v}_2$. This is done by taking the product of transpose of the matrix **N** and **V**, and the resultant matrix denoted as **R**

$$\mathbf{R}_{3\times m} = \mathbf{V}^{\mathrm{T}}{}_{3\times 3} \cdot \mathbf{N}^{\mathrm{T}}{}_{3\times m} \quad (4)$$

Each element in first row of **R** will give the *x*-coordinates and the each element in second row will give the *y*-coordinates, combined together in pair as (x', y'). Now the equation of circle in 2-dimension is

$$(x - c_1)^2 + (y - c_2)^2 = r^2$$

$$\text{or,} \quad 2xc_1 + 2yc_2 + k_3 = x^2 + y^2 \quad (5)$$

where, $k_3 \equiv r^2 - c_1^2 - c_2^2$

The set of *m* data points *(x, y)* will result in *m* number of equations in three unknowns c_1, c_2 and k_3.

$$\begin{bmatrix} 2x'_1 & 2x'_1 & 1 \\ \vdots & \vdots & \vdots \\ 2x'_m & 2x'_m & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} x'^2_1 + y'^2_1 \\ \vdots \\ x'^2_m + y'^2_m \end{bmatrix} \quad (6)$$

Equation (6) is a set of linear equation of form **Ax**=**b**. Moreover, **A** is $m\times 3$ matrix, **x** is the 3×1 column, **b** are defined as**,**

$$\mathbf{A} \equiv \begin{bmatrix} 2x'_1 & 2x'_1 & 1 \\ \vdots & \vdots & \vdots \\ 2x'_m & 2x'_m & 1 \end{bmatrix}; \quad \mathbf{x} \equiv \begin{bmatrix} c_1 \\ c_2 \\ k_3 \end{bmatrix}$$

$$\text{and} \quad \mathbf{b} \equiv \begin{bmatrix} x'^2_1 + y'^2_1 \\ \vdots \\ x'^2_m + y'^2_m \end{bmatrix} \quad (7)$$

The least square fit for the center of the circle [15] is found using pseudoinverse of matrix **A**,

$$\mathbf{x} = ((\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}})\mathbf{b} \quad (8)$$

The first two element of **x** will give coordinate of center (c_1, c_2) and the third element gives radius r,

$$r = \sqrt{c_1^2 + c_2^2 - k_3} \quad (9)$$

The coordinates of the center (c_1, c_2) were then translated back into the space of 3D data points. The coordinates of the centroid were then added to obtain the center (*C)* in the actual space of data as $[c_x, c_y, c_z]^{\mathrm{T}}$.

III. METHODOLOGY

This section explains the methods used to find the DH parameters of an industrial robot KUKA KR5. These parameters are either unknown or there is a mismatch between the given kinematic parameters with the actual one specified by the manufacturers. The work flow is presented in Fig. 5.

A. *Data Acquisition*

The input required for the extraction of DH parameters of a robot is the position of a point on the end-effector frame with respect to the measurement frame. In the identification of DH parameters of a robot under study, one needs to get these positions from an independent measurement device whose accuracy should be more than that of the robot. However, here the robot's controller has been used to get the end-effectors position to test the proposed algorithm. In future, measurement devices will be used. For the measurement, a point on the end-effector of KUKA KR5 was assigned as tool frame in its controller. The range of motion given to each revolute joint was kept arbitrarily between 90 and 30 degrees for two sets of

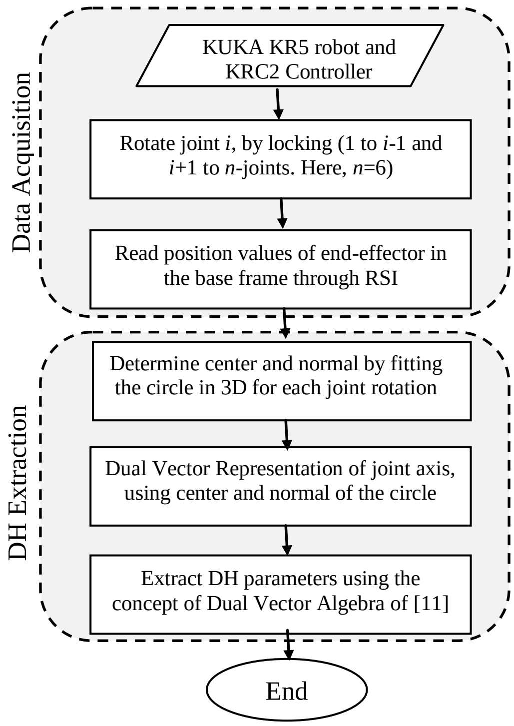


Fig. 5. Flowchart for finding the DH parameters

readings. From the first joint to the last, each joint was rotated in a counter clock wise direction, locking all the other joints. The coordinates of the point on end-effector were measured in the base frame of the robot.

Range of motion given to each joint is reported in Table II. The home position of the robot was kept at (A1 90° A2 -90°, A3 90°, A40°, A5 90°, A6 0°), where 'A' represents axis and the corresponding number denotes the angle. The measurements were taken at a low speed (10% of the maximum speed of the robot i.e. 2 m/s) as the accuracy of measurement is higher at low speed.

TABLE II. RANGE OF MOTION PROVIDED TO EACH JOINT

Axis	90 Degree Range		30 Degree Range	
Joint No.	θ_{min}	θ_{max}	θ_{min}	θ_{max}
1	-45	45	-15	15
2	-60	-130	-60	-90
3	120	30	120	90
4	90	0	90	60
5	90	0	90	60
6	0	90	0	30

An add-on to standard KUKA Robot Language (KRL), Robot Sensory Interface (RSI) of KUKA KR5 [16] was used to obtain the coordinates of the end-effector in real time. This allows one to obtain different run time parameters like joint angles, end-effector position, gear torque, motor current etc. with time. The PC was connected to KUKA controller using a standard Ethernet cable. The first three parameters of RSI object ST_ACTPOS returns the current position of the calibrated tool tip on the end-effector. This is connected to the RSI object ST_MONITOR using ST_NEWLINK command. Once the monitoring was started with ST_SETPARAM and RSI is started using ST_ON command, the joint was moved in desired path.

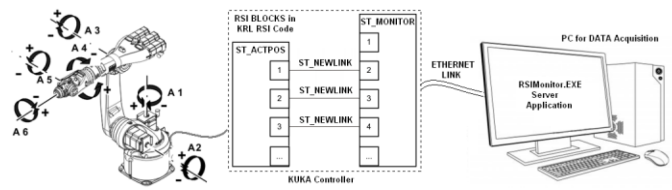


Fig. 6. Data acquisition system

The object ST_MONITOR sends the values of the end-effector position to the PC, where a server application for monitoring, namely, 'RSIMonitor.exe' was executed. The application then was used to record the end-effector position with time in seconds.

The measurement of all the six joints were taken from the RSI. The conditions taken into account for the measurement of target points were:

- The target point was selected on the edge of the end-effector and referred as Tool Frame in the robot controller.
- The measured target points were unifromly distributed in the upper and lower limits of the joint specified. Note that the target point should not be along the axis of the 6^{th} joint, else the circle corresponding to 6^{th} joint would be a point instead of a circle which is desired.
- The other joints were locked in the home poisiton except the one joint which has been given a specified range of motion mentioned in Table II.
- The standard steps in the measurement of target point between the time interval of given joint movement was set in the RSI.

B. *Extraction of DH Parameters*

Fig. 7 shows the fitted circles and normals for all the joints.

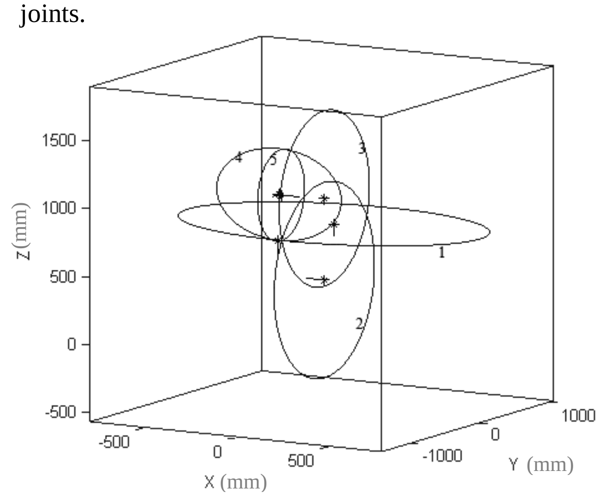


Fig. 7. Fitted circle for each joint motion

TABLE III. SPECIFIED AND IDENTIFIED DH PARAMETERS OF KUKA KR5

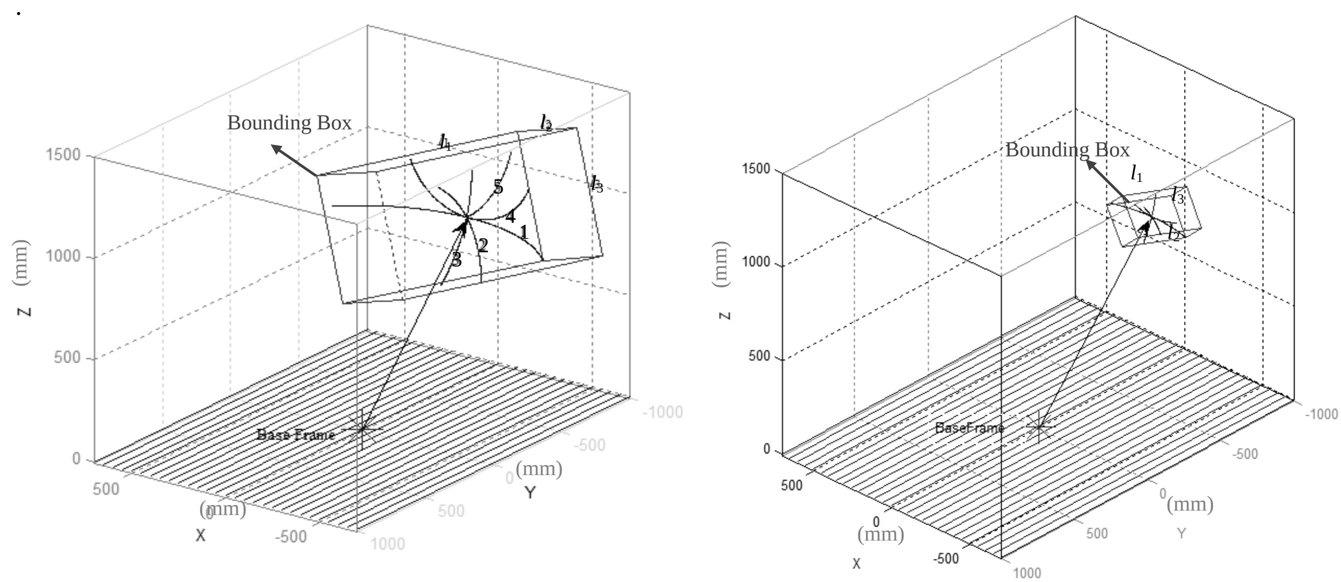
Joint No.	Joint Offset (*b*) (mm)			Link Length (*a*) (mm)			Twist Angle (*α*) (degree)		
	Specified	Identified (with range)		Specified	Identified (with range)		Specified	Identified (with range)	
		90°	30°		90°	30°		90°	30°
1	400	400.011	400.022	180	179.992	180.01	90	90	90
2	0	0	0	600	599.998	600.011	0	0	0
3	0	0.002	0.062	120	119.991	119.904	90	90	90.005
4	620	620.042	620.081	0	0	0	90	90	90.005
5	0	0.001	0.019	0	0	0	90	90	89.994
6	0	0	0	0	0	0	0	0	0

The figure also indicates the range of motion that one joint can undergo without the mechanical or software limit. Note that full circles are drawn rather than the circular arc for the measured points. It is to be noted here that the sixth joint center and normal is not visible as the circle traced by it has small radius compared to the scale of other circle.

The set of Cartesian data points were obtained from the measurements with respect to the base frame of the robot in the RSI. Note that the measured points lay on a circle in the 3D space, since only a single revolute joint was provided motion. In Fig. 3, the frame (*XYZ*) is the sensor coordinate frame in which the readings were taken. The center and normal of the circle traced by the rotation of each joint were determined using Singular Value Decomposition and Least Square Fit method as explained in Section II(C). The DH parameters were then extracted using the dual vector algorithm given in [11-12].

C. Results and Discussion

The performance of the proposed methodology was tested by changing the range of motion given to the joints and evaluating the DH parameters again. Table III lists the DH parameters specified by the robot manufacturers and identified using 90° and 30° ranges, the identified joint offset and link length readings in millimeters for the two ranges of motion (90 and 30 degrees) shows very little variations, there by confirms the robustness of the proposed algorithm.
Oriented bounding box, i.e., a box with minimum value which has all the measured points, was obtained for both ranges of joint angle motion. Table IV lists the length l_1, breadth l_2, height l_3 and the orientation of the bounding box. These can be utilized for deciding the positioning of the measuring device near the end-effector. The orientation of the bounding box can be useful in deciding the orientation of the measurement devices. Keeping the devices parallel to this orientation will allow to measure accurately as the total range of motion provided to each of the robotic arm, can be covered from this position.



a) Bounding box for 90° range of motion of each joint

b) Bounding box for 30° range of motion of each joint

Fig. 8. Range of motion provided to each joint.

TABLE IV. BOUNDING BOX DIMENSION FOR DIFFERENT RANGE OF MOTION PROVIDED TO EACH OF THE JOINTS

	Range of motion given to each joint (Degrees)	
	90°	30°
l_1 (mm)	1032.48	421.351
l_2(mm)	599.696	114.590
l_3(mm)	489.250	208.447
Volume (m^3)	0.312	0.010
Orientation Matrix	$\begin{bmatrix} -0.072 & -0.912 & 0.403 \\ 0.953 & -0.181 & -0.240 \\ 0.292 & 0.366 & 0.883 \end{bmatrix}$	$\begin{bmatrix} -0.897 & -0.435 & 0.071 \\ -0.069 & -0.019 & -0.997 \\ 0.435 & -0.899 & -0.013 \end{bmatrix}$

IV. CONCLUSION

Identification of the DH parameters of an industrial robot is proposed by using the end-effector's Cartesian positions with respect to the base frame. Robot sensory interface was used instead of an external measurement device to take the end-effector position readings. With these measurements circles can be defined which were then used for the identification of DH parameters. This proposed method showed that a small range of the end-effector did not change the identified results for the DH parameters. Hence the robustness of the algorithm has been established. In future, the identification will be carried out using suitable external measurement devices.

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