A Unified Approach to Space Robot Kinematics

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***Abstract*— An essential step in deriving kinematic models of free-flying space robots, consisting of a free-base and a manipulator mounted on it, is to write the total momenta of the system at hand. The momenta are, usually, expressed as the functions of the velocities of a *preselected* body that belongs to the robot, e.g., the free-base. In this paper, *no* preselection is recommended. On the contrary, the total momenta are expressed as the functions of the velocities of an *arbitrary* body of the space robot, namely, the *primary body* (PB). The identity of the PB, unlike the conventional approaches, need not be known at this stage. Therefore, the generalized expressions for the total momenta are obtained. The resulting expressions can explain the existing kinematic models and how they affect the efficiencies of the associated control algorithms. Based on the proposed approach, it is shown that if the end-effector motion is the only concern, as desired in kinematic control, it should be selected as the PB. This leads to the most efficient algorithms.**

I. Introduction

KINEMATICS of the *free-flying* or *free-floating* space robots, comprising of a free-base, e.g., a spacecraft and a manipulator mounted on it, is the subject of this paper. In space robotics, the term *free-flying* or *free-floating* implies that the space robot under study is controlled by its joint actuators only. As a result, the disturbance of the spacecraft due to the reaction moments and forces at the interface of the spacecraft and the manipulator is allowed. In order to achieve such control, either the momenta conservation principle (MCP) or the constraints on the total momenta must be satisfied. The latter is, however, essential while the space robot is interacting with its environment, i.e., subjected to external moments and forces [1].

Now, in relation to the control of free-flying space robots, it has been pointed out in [2] that any control algorithm suitable for the fixed-base manipulators also can be employed in the control of free-flying space manipulators, with the additional conditions of estimating or measuring a spacecraft's orientation and of avoiding dynamic singularities. Therefore, a "resolved motion rate control" of the space robot, as proposed in [3] using the *Generalized Jacobian Matrix* (GJM), was possible. The GJM is an extension of the Jacobian matrix for the fixed-base manipulators. The matrix is also derived by others, e.g., in [1] and [4]. In [5], however, the GJM, which is originally defined with respect to the spacecraft [3], is derived with respect to other bodies of the space robot as well. Hence, the GJM's with respect to the end-effector and any other rigid body in the kinematic chain of the space robot can be defined. The definitions, nevertheless, do not provide any additional advantage over the original GJM.

In all those attempts [1], [3]–[5], it has been noticed that, in order to derive the kinematic model of the free-flying space robots, first, a body is *selected*, e.g., the spacecraft in [3] or the whole system as the composite body in [5]. Then, the velocities of the *preselected* body are used to express the total momenta of the system at hand. Finally, the kinematic model is derived. The disadvantage of this approach is that no generalized expression for the total momenta is available, which could suggest a suitable choice of the body that will satisfy a desired requirement, e.g., an efficient control algorithm. Hence, different expressions for the total momenta in terms of the velocities of different preselected bodies have been emerged, probably, in search for an efficient kinematic algorithm. Such attempts were attributed to the derivation of the total momenta from the definition given by (4). The explanation appears after the equation.

In order to overcome the above problem, the total momenta of the space robot at hand are derived here from its total kinetic energy. This allows to express the total momenta in terms of the velocities of an *arbitrary* body, which is called the *primary body* (PB). The identity of the PB need not be known at this stage. It means that *no* preselection is required. Moreover, to obtain the kinematic model, first, the total momenta are written as the functions of the velocities of the PB. Then, the PB is selected. Finally, the kinematic model is attempted. In effect, the first two conventional steps are interchanged and the concept of PB is introduced. The approach has the following features.

i) The generalized expressions for the total momenta in terms of the velocities of the PB are obtained.
ii) Since the PB is arbitrary, a choice of the spacecraft as the PB leads to the kinematic model based on the *generalized Jacobian matrix* (GJM) [3], whereas the composite body comprising of all the rigid bodies of the space robot as the PB explains the model reported in [5].
iii) For kinematic analyses, where the relations between the end-effector and the joint motions are of interest, it is shown in this paper that the choice of the end-effector as the PB, among other choices, will lead to the kinematic model that results in the *most* efficient algorithms. It is pointed out after (24) and (25).

Interestingly, a model similar to the one, mentioned in iii), has been also derived in a separate independent research [6] where, however, no reason is given for its efficiency. In addition, the

Manuscript received March 14, 1994; revised December 1, 1995. This paper was recommended for publication by Associate Editor B. Siciliano and Editor S. E. Salcudean upon evaluation of reviewers' comments.

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Publisher Item Identifier S 1042-296X(96)00495-8.

point about the *most* efficiency could not be made because their approach is based on the conventional steps that lack generality.

This paper is organized as follows: In Section II, the generalized expressions for the total momenta are derived as the *extended total momentum* (ETM) and the concept of PB is introduced. Section III shows how the GJM can be derived using the proposed approach. The kinematic model using the end-effector as the PB is also presented there. In Section IV, the efficiencies of the direct and inverse kinematic algorithms, based on the proposed model, are compared. Finally, the contribution of the paper is discussed in Section V.

II. Extended Total Momemtum and the Primary Body

The space robot under study is assumed to be consisting of $n+1$ rigid bodies, denoted in Fig. 1 by $\#0, \cdots, \#n$, connected by n revolute joints located at $O_1, \cdots, O_n$. Referring to the motion of the ith body, some definitions are introduced as follows:

$\boldsymbol{t}_i$: the six-dimensional vector of *twist*, which is defined as

$$\boldsymbol{t}_i \equiv \begin{bmatrix} \boldsymbol{\omega}_i \\ \dot{\boldsymbol{c}}_i \end{bmatrix} \tag{1}$$

where $\boldsymbol{\omega}_i$ and $\dot{\boldsymbol{c}}_i$ are the three-dimensional vectors representing the angular velocity and the velocity of the mass center of the ith body, C_i, respectively. Note that, in order to define the twist of a rigid body, any point other than C_i may be chosen also. This is done for the twist of the end-effector, as in (22).

$\boldsymbol{M}_i$: the 6×6 *extended mass* matrix of the ith body, i.e.,

$$\boldsymbol{M}_i \equiv \begin{bmatrix} \boldsymbol{I}_i & \mathbf{o} \\ \mathbf{o} & m_i \mathbf{1} \end{bmatrix} \tag{2}$$

where m_i and $\boldsymbol{I}_i$ are the mass of the ith body and its 3×3 inertia tensor about C_i, respectively, whereas $\mathbf{1}$ and $\mathbf{o}$ are the 3×3 identity and zero tensors, respectively.

$\boldsymbol{h}$: the six-dimensional *extended total momentum* (ETM) of the system, given by

$$\boldsymbol{h} \equiv \begin{bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{m} \end{bmatrix} \tag{3}$$

where $\boldsymbol{\alpha}$ and $\boldsymbol{m}$ are the three-dimensional vectors of total angular momentum about an inertially fixed point, say, O of Fig. 1, and the total linear momentum, respectively. They are defined as

$$\boldsymbol{\alpha} \equiv \sum_{i=0}^{n} (\boldsymbol{I}_i \boldsymbol{\omega}_i + \boldsymbol{c}_i \times m_i \dot{\boldsymbol{c}}_i) \quad \text{and} \quad \boldsymbol{m} \equiv \sum_{i=0}^{n} m_i \dot{\boldsymbol{c}}_i \tag{4}$$

in which if $\boldsymbol{\alpha}$ is derived about a moving point S, it will be denoted by $\boldsymbol{\alpha}_s$. The subscript, "s," will be explained as it appears. Consequently, the ETM, $\boldsymbol{h}$, will be represented as $\boldsymbol{h}_s$. The expression for the total linear momentum that does not depend on any such point will, however, always be denoted by $\boldsymbol{m}$.

Note that, if the definition given by (4) is used in an attempt to derive the kinematic model of the free-flying space robot, there is a natural tendency to *select* a body first and express the velocities of the other bodies in terms of the velocities of the *selected* body, as reflected in [1] and [3]–[5].

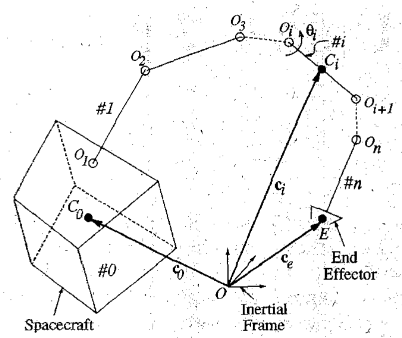

Fig. 1. A schematic diagram of a space robot.

Therefore, in this paper, the ETM is obtained from the total kinetic energy of the system at hand. In this approach, no selection is necessary at this stage. The derivation is, initially, explained for the motion of one body, say, i. For the ith rigid body, the kinematic energy, T_i, is given by

$$T_i = \frac{1}{2} \boldsymbol{t}_i^T \boldsymbol{M}_i \boldsymbol{t}_i \tag{5}$$

where $\boldsymbol{t}_i$ and $\boldsymbol{M}_i$ are, respectively, defined in (1) and (2). From (5), it follows that

$$\frac{\partial T_i}{\partial \boldsymbol{t}_i} \equiv \boldsymbol{M}_i \boldsymbol{t}_i. \tag{6}$$

What the right hand side of (6) implies is that the term, $\boldsymbol{M}_i \boldsymbol{t}_i$, is the six-dimensional vector consisting of the components of the angular momentum of the ith body about C_i and the body's linear momentum. In other words, the ETM of the ith body is obtained.

Extending the concept of the ETM of the ith body to the whole system, a definition for the total momenta or the ETM is introduced as

$$\boldsymbol{h}_p \equiv \frac{\partial T}{\partial \boldsymbol{t}_p} \tag{7}$$

where T is total kinematic energy of the space robot consisting of $n+1$ rigid bodies, Fig. 1, which can be written as

$$T = \frac{1}{2} \sum_{i=0}^{n} \boldsymbol{t}_i^T \boldsymbol{M}_i \boldsymbol{t}_i \equiv \frac{1}{2} \boldsymbol{t}^T \boldsymbol{M} \boldsymbol{t} \tag{8}$$

where the $6(n+1)$-dimensional vector, $\boldsymbol{t}$, and the $6(n+1) \times 6(n+1)$ matrix, $\boldsymbol{M}$, are as follows:

$$\boldsymbol{t} \equiv \left[\boldsymbol{t}_0^T, \cdots, \boldsymbol{t}_n^T\right]^T \text{and } \boldsymbol{M} \equiv \text{diag}(\boldsymbol{M}_0, \cdots, \boldsymbol{M}_n). \tag{9}$$

It is pointed out here that, for the system having different topology other than the one shown in Fig. 1, e.g, a parallel space robot, (8) should be calculated accordingly. Now, referring to the definition of the total momenta given by (7), which will lead to the generalized expressions for the total momenta, the following points are indicated.

a) The angular momentum associated to $\boldsymbol{h}_p$ is calculated about the mass center of the body having twist $\boldsymbol{t}_p$. The body is denoted by $\mathcal{P}$ and given the name *primary body*

(PB). Note that, if a point other than the mass center is used to define $\boldsymbol{t}_p$, e.g., $\boldsymbol{t}_e$ of (22), the associated ETM, $\boldsymbol{h}_p$, would imply that its angular momentum is calculated about the point specified in defining $\boldsymbol{t}_p$.

b) Equation (7) is defined with respect to the twist of a rigid body whose choice is not limited. Hence, it could be any body that belongs to the kinematic chain of the system or the whole system as the composite body [5].

From a), since the angular momentum of the ETM, (7), is calculated about a moving point, (7) is not suitable for the development of a control algorithm, where it is always desired to find the total momenta in an inertial frame, i.e., $\boldsymbol{h}$ of (3). This implies that the angular momentum be evaluated about an inertially fixed point, say, O of Fig. 1. The problem is resolved with the aid of the *translation theorem* [7], which states that

$$\boldsymbol{h} = \boldsymbol{B}_p \boldsymbol{h}_p \tag{10}$$

where the 6×6 matrix, $\boldsymbol{B}_p$, is defined as

$$\boldsymbol{B}_p \equiv \begin{bmatrix} \mathbf{1} & \boldsymbol{C}_p \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \tag{11}$$

in which $\boldsymbol{C}_p$ is the 3×3 cross-product tensor associated to the vector, $\boldsymbol{c}_p$, that denotes the position of the mass center of body $\mathcal{P}$, C_p, as indicated in Fig. 1 for the ith body, $\boldsymbol{c}_i$. Moreover, the cross-product tensor $\boldsymbol{Z}$ associated to the vector $\boldsymbol{z}$ is defined as

$$\boldsymbol{Z} \equiv \boldsymbol{z} \times \mathbf{1} \equiv \frac{\partial(\boldsymbol{z} \times \boldsymbol{x})}{\partial \boldsymbol{x}} \tag{12}$$

for an arbitrary three-dimensional vector $\boldsymbol{x}$. Now, the substitution of (8) into (7) yields

$$\boldsymbol{h}_p \equiv \left(\frac{\partial \boldsymbol{t}}{\partial \boldsymbol{t}_p}\right)^T \boldsymbol{M}\boldsymbol{t} \tag{13}$$

where to obtain the expression, $\partial \boldsymbol{t}/\partial \boldsymbol{t}_p$, the generalized twist, $\boldsymbol{t}$ of (9), must be obtained as a function of $\boldsymbol{t}_p$, i.e., the twists of all the rigid bodies in the chain must be written in terms of $\boldsymbol{t}_p$. Hence, the name *Primary Body*, given to the body $\mathcal{P}$, is justified. Using the kinematic constraints, vector $\boldsymbol{t}$ is expressed as

$$\boldsymbol{t} = \boldsymbol{T}_p \boldsymbol{t}_p + \boldsymbol{t}' \tag{14}$$

in which $\boldsymbol{T}_p$ is the $6(n+1) \times 6$ matrix associated to $\boldsymbol{t}_p$, and $\boldsymbol{t}'$ represents the $6(n+1)$-dimensional vector that is a function of the joint motions. The expression of $\boldsymbol{t}'$ depends on the types of couplings present in the kinematic chain, e.g., a revolute or a prismatic pair. The term, $\partial \boldsymbol{t}/\partial \boldsymbol{t}_p$, is next obtained from (14) as

$$\frac{\partial \boldsymbol{t}}{\partial \boldsymbol{t}_p} \equiv \boldsymbol{T}_p \tag{15}$$

and, substituting (15) into (13), $\boldsymbol{h}_p$ is given by

$$\boldsymbol{h}_p = \boldsymbol{T}_p^T \boldsymbol{M}\boldsymbol{t}. \tag{16}$$

Finally, the ETM, $\boldsymbol{h}$ of (10), is derived from (14) and (16) as

$$\boldsymbol{h} = \boldsymbol{B}_p \boldsymbol{T}_p^T \boldsymbol{M}(\boldsymbol{T}_p \boldsymbol{t}_p + \boldsymbol{t}'). \tag{17}$$

Note in (17) that the ETM, $\boldsymbol{h}$, unlike in the conventoinal approaches, is derived here as a function of the velocities of an *arbitrary* body, namely, the PB. The identity of the PB need not be known at this stage, i.e., whether the PB is the spacecraft or the end-effector is not essential here. However, a choice about the PB must be made before writing the kinematic equations for control algorithms. The criteria depend on the primary objective of the kinematic analyses, as shown next.

III. Kinematic Equations

As to show how the concept of PB can be used in deriving the kinematic model, two sets of equations using the spacecraft and the end-effector as the PB are derived. The former leads to the definition of the GJM [3], as done in Section III-A.

A. Derivation of the Generalized Jacobian Matrix

Here the interest is to examine the reaction effect of the manipulator on its base [3], i.e., the motion of the spacecraft must be known. Hence, the spacecraft is selected as the PB. The generalized twist of the space robot, $\boldsymbol{t}$ of (14), can, therefore, be expressed as

$$\boldsymbol{t} = \boldsymbol{T}_0 \boldsymbol{t}_0 + \boldsymbol{T}_m \dot{\boldsymbol{\theta}} \tag{18}$$

where $\boldsymbol{t}_0$ is the twist of the spacecraft, body #0 of Fig. 1, as defined in (1), and $\dot{\boldsymbol{\theta}}$ is the n-dimensional vector of the joint rates, i.e.,

$$\dot{\boldsymbol{\theta}} \equiv [\dot{\theta}_1, \cdots, \dot{\theta}_n]^T \tag{19}$$

where the scalar, $\dot{\theta}_i$, for $i = 1, \cdots, n$, being the angular displacement of the revolute joint located at O_i, Fig. 1. Moreover, $\boldsymbol{T}_0$ and $\boldsymbol{T}_m$ are the $6(n+1) \times 6$ and the $6(n+1) \times n$ matrices, respectively. Comparison of (14) and (18) shows that $\boldsymbol{t}' = \boldsymbol{T}_m \dot{\boldsymbol{\theta}}$. Using (18), the ETM, $\boldsymbol{h}$ of (17), is then obtained as

$$\boldsymbol{h} = \boldsymbol{B}_0(\boldsymbol{I}_0 \boldsymbol{t}_0 + \boldsymbol{I}_m \dot{\boldsymbol{\theta}}) \tag{20}$$

where the 6×6 symmetric matrix, $\boldsymbol{I}_0$, and the $6 \times n$ matrix, $\boldsymbol{I}_m$, are given by

$$\boldsymbol{I}_0 \equiv \boldsymbol{T}_0^T \boldsymbol{M} \boldsymbol{T}_0 \quad \text{and} \quad \boldsymbol{I}_m \equiv \boldsymbol{T}_0^T \boldsymbol{M} \boldsymbol{T}_m. \tag{21}$$

However, the 6×6 matrix, $\boldsymbol{B}_0$, is defined similar to $\boldsymbol{B}_p$, (11), associated to the position of the spacecraft, $\boldsymbol{c}_0$ of Fig. 1. Equation (20) represents a relation between the twist of the spacecraft and the joint rates of the manipulator. But, for the kinematic analyses, relations between the twist of the end-effector and the joint rates are desired. Thus, additional equations are required, which are written from the kinematic constraints as

$$\boldsymbol{t}_e = \boldsymbol{J}_0 \boldsymbol{t}_0 + \boldsymbol{J}_m \dot{\boldsymbol{\theta}} \tag{22}$$

where $\boldsymbol{t}_e$ is the twist of the end-effector, i.e., $\boldsymbol{t}_e \equiv [\boldsymbol{\omega}_e^T, \dot{\boldsymbol{c}}_e^T]^T$, $\boldsymbol{\omega}_e$ and $\dot{\boldsymbol{c}}_e$ being the angular velocity of the end-effector and the velocity of point E on the end-effector, Fig. 1, respectively. The terms, $\boldsymbol{J}_0$ and $\boldsymbol{J}_m$, are the 6×6 and the $6 \times n$ matrices, respectively. From (20) and (22), the desired kinematic model is derived as

$$\boldsymbol{t}_e = \boldsymbol{J}^* \dot{\boldsymbol{\theta}} + \boldsymbol{H}^* \boldsymbol{h} \tag{23}$$

in which the $6 \times n$ matrix, $\boldsymbol{J}^*$, and the 6×6 matrix, $\boldsymbol{H}^*$, are as follows:

$$\boldsymbol{J}^* \equiv \boldsymbol{J}_m - \boldsymbol{J}_0 \boldsymbol{I}_0^{-1} \boldsymbol{I}_m \quad \text{and} \quad \boldsymbol{H}^* \equiv \boldsymbol{J}_0 \boldsymbol{I}_0^{-1} \boldsymbol{B}_0^{-1}. \tag{24}$$

Since $\boldsymbol{I}_0$ is a symmetric positive definite (SPD) matrix, its inverse is always defined. Also, from (11), it is clear that $\boldsymbol{B}_0^{-1}$ exists. Matrix $\boldsymbol{J}^*$ is the so called *generalized Jacobian matrix* (GJM), as originally derived in [3], and obtained here using the spacecraft as the PB.

Note that the GJM approach is essential while, in addition to the end-effector control, the motion of the spacecraft is also desired for some other purposes, for example, attitude control problem of the space robots. However, if the end-effector motion is the only concern, (17) suggests that the end-effector should be chosen as the PB. This avoids the calculations of the additional step of the GJM, (22), and directly leads to the kinematic model, resulting in more efficient algorithms. This is done next.

B. Kinematic Model Using the End-Effector as the Primary Body

A kinematic model based on the end-effector as the PB is proposed. For this, the generalized twist, $\boldsymbol{t}$, is derived, similar to (18), as

$$\boldsymbol{t} = \boldsymbol{T}_e \boldsymbol{t}_e + \hat{\boldsymbol{T}}_m \dot{\boldsymbol{\theta}} \tag{25}$$

where $\boldsymbol{t}_e$ is defined after (22) and $\dot{\boldsymbol{\theta}}$ is given by (19). Also, $\boldsymbol{T}_e$ and $\hat{\boldsymbol{T}}_m$ are the $6(n+1) \times 6$ and the $6(n+1) \times n$ matrices, respectively. The ETM, $\boldsymbol{h}$, using the end-effector as the PB is then derived, similar to (20), as

$$\boldsymbol{h} = \boldsymbol{B}_e(\boldsymbol{I}_e \boldsymbol{t}_e + \hat{\boldsymbol{I}}_m \dot{\boldsymbol{\theta}}) \tag{26}$$

where the 6×6 matrix, $\boldsymbol{B}_e$, is associated to $\boldsymbol{c}_e$ and defined similar to (11), whereas $\boldsymbol{I}_e$ and $\hat{\boldsymbol{I}}_m$, the 6×6 and the $6 \times n$ matrices, respectively, are as follows:

$$\boldsymbol{I}_e \equiv \boldsymbol{T}_e^T \boldsymbol{M} \boldsymbol{T}_e \quad \text{and} \quad \hat{\boldsymbol{I}}_m \equiv \boldsymbol{T}_e^T \boldsymbol{M} \hat{\boldsymbol{T}}_m. \tag{27}$$

In contrast to (20), (26) directly leads to the kinematic model that relates the twist of the end-effector to the joint rates, i.e.,

$$\boldsymbol{t}_e = \boldsymbol{J} \dot{\boldsymbol{\theta}} + \boldsymbol{H} \boldsymbol{h} \tag{28}$$

where $\boldsymbol{J}$ and $\boldsymbol{H}$ are the $6 \times n$ Jacobian matrix and the 6×6 matrix associated to $\boldsymbol{h}$, respectively. They are given by

$$\boldsymbol{J} \equiv -\boldsymbol{I}_e^{-1} \hat{\boldsymbol{I}}_m \quad \text{and} \quad \boldsymbol{H} \equiv \boldsymbol{I}_e^{-1} \boldsymbol{B}_e^{-1} \tag{29}$$

in which $\boldsymbol{I}_e^{-1}$ exists, since it is a SPD matrix. Comparing (24) and (29), it is now obvious that the expression of matrix $\boldsymbol{J}$ is less complex than that of the GJM, $\boldsymbol{J}^*$. In fact, as evident from (17), no other choice than the end-effector as the PB would provide the least complex expression for the Jacobian matrix associated to the kinematic model of the free-flying space robots. A kinematic model similar to (28) is also reported in [6], but failed to state the reason for its efficiency and could not point out that the choice is the best.

TABLE I
COMPUTATIONAL COMPLEXITIES

Algorithm	DOF	Multiplications (M)	Additions (A)	Additional
(A) Calculation of the GJM, $\mathbf{J}^*$, and $\mathbf{J}$.				
$\mathbf{J}^*$ in [4]	n	$163n + 72$	$145n + 48$	nil
$\mathbf{J}^*$ in [5]	n	$162n + 87$	$132n + 59$	nil
Proposed $\mathbf{J}$	n	$113n - 11$	$96n - 11$	nil
(B) Computation of the joint rates, $\dot{\boldsymbol{\theta}}$.				
In [4]	n	$163n + 72$	$145n + 48$	$+c_s$
In [5]	n	$162n + 87$	$132n + 59$	$+c_s$
In [6]	n	$165n + 273$	$130n + 190$	$+c_s$
Proposed	n	$93n + 12$	$84n + 6$	$+c_s$

c_s: cannot be expressed in terms of n.

IV. COMPLEXITY IN DIRECT AND INVERSE KINEMATICS

While (23) and (28) are required for the kinematic analyses of the free-flying space robots under no momenta conservation, they are simplified for the following assumptions:

- the momenta conservation principle (MCP) is applicable, i.e., $\mathrm{h} = \mathbf{constant}$;
- the space robot is initially at rest, i.e., $\mathrm{h} = \mathbf{0}$.

With the above two assumptions, (23) and (28), respectively, reduce to

$$\boldsymbol{t}_e = \boldsymbol{J}^* \dot{\boldsymbol{\theta}} \quad \text{and} \quad \boldsymbol{t}_e = \boldsymbol{J} \dot{\boldsymbol{\theta}} \tag{30}$$

where the GJM, $\boldsymbol{J}^*$, and the Jacobian matrix, $\boldsymbol{J}$, are given by (24) and (29), respectively. A point regarding the computation of the matrices, $\boldsymbol{I}_0$, $\boldsymbol{I}_m$ and $\boldsymbol{I}_e$, $\hat{\boldsymbol{I}}_m$, required to compute $\boldsymbol{J}^*$ and $\boldsymbol{J}$, respectively, is made clear here. If the matrices are calculated in a straightforward manner, an order n^2 algorithm will be obtained, as done in [1] and verified by the author, whereas a concept of the composite bodies, as done in [4]–[6], will result in a recursive algorithm of order n. In this paper, the matrices, $\boldsymbol{I}_e$ and $\hat{\boldsymbol{I}}_m$, associated to the proposed model are calculated using the composite body concept.

Now, the computational complexities in direct and inverse kinematics are compared. They are based on the above two assumptions for which some results are available.

A) *Direct Kinematics*: Direct kinematics is defined, given the joint variables calculate the end-effector motion. This can be done using either of the equations appearing in (30), whose complexities are evident from the computational requirement of the associated Jacobian matrices, namely, the GJM, $\boldsymbol{J}^*$ of (24), and $\boldsymbol{J}$ of (29). The complexities are shown in Table I(a).

B) *Inverse Kinematics*: Inverse kinematics, on the other hand, is used to find the joint variables from known end-effector motion, i.e., either equation in (30) must be solved. The computational complexity to find $\dot{\boldsymbol{\theta}}$ is given in Table I(b), where c_s is associated to the solution of the set of six linear algebraic equations in n unknowns.

V. CONCLUSION

From Table I, it is clear that the proposed kinematic model based on the end-effector as the PB results in most efficient direct and inverse kinematic algorithms. The efficiencies are attributed to:

i) the ability to choose the end-effector, as the PB, from the generalized expression of the total momenta, (17);

ii) the efficient calculation of different terms, particularly, those containing the orientation matrices. For example, to find an inertia tensor of a body, either individual or composite, from its representation in one particular frame to the successive frame, requires multiplications of three 3×3 matrices. In the proposed algorithm, they are efficiently done only with 16 multiplications (M) and 17 additions (A). It was possible due to the orthogonality of the orientation matrices and the symmetricity of the inertia tensors.

Note in Table I(B), the increased efficiency of the proposed algorithm for inverse kinematics, compared to those in [4], [5]. The reason is: in the GJM based model, in order to find $\dot{\boldsymbol{\theta}}$, the complex expression of $\boldsymbol{J}^*$, (24), must be evaluated, whereas, in the present scheme, the explicit calculation of $\boldsymbol{J}$ is not required, as $\dot{\boldsymbol{\theta}}$ is solved from (29) and (30) as, $\hat{\boldsymbol{I}}_m\dot{\boldsymbol{\theta}} = -\boldsymbol{I}_e\boldsymbol{t}_e$, where the vector, $\boldsymbol{I}_e\boldsymbol{t}_e$, is the input. It is emphasized here that, besides the efficiency, the derivation of the total momenta of the space robot from its total kinetic energy, (17), and the introduction of the concept of PB have the following characteristics. The proposed approach:

i) explains all the existing models, as discussed in Section I and shown in Section III-A for the GJM based model;

ii) clarifies why the choice of the end-effector as the PB, compared to any other choice, leads to the *most* efficient algorithms. The reasons are given after (29), which could not be pointed out in any other literature.

Hence, the concept of PB gives a significant insight to the kinematics of the free-flying space robots and may be considered as a useful tool to study space robotics.

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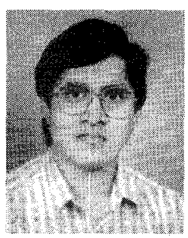

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