The Kinematic Design of a 3-dof Isotropic Mobile Robot

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Abstract

Automatic guided vehicles with omnidirectional wheels have three degrees of freedom (DOF), their full mobility being obtained by virtue of the free rollers around the periphery of the wheels. The choice of the orientation of the roller axes with respect to the wheel axis, along with the wheel orientations with respect to the platform, the number of wheels, etc., are design issues considered here. A design approach based on the isotropy of the underlying Jacobian matrices is reported in this paper.

1 Introduction

Most commercially available automatic guided vehicles (AGVs) are supplied with conventional wheels, i.e., wheels that consist basically of a single disk rotating about one axis, like those used in automobiles, landing gears, bicycles, etc. [1]. Unless complicated mechanisms for reorienting the planes of such wheels are used, these wheels cannot produce but 2-DOF motion [2]. The invention of the *Mekanum wheel* [3] allows for a 3-DOF mobility in a vehicle without additional mechanisms. In the literature, Mekanum wheels are also referred to as *omnidirectional* wheels [2] and *ilonators*. An omnidirectional wheel consists of a wheel hub and rollers that are mounted on the hub. Moreover, the axes of the rollers make an angle α with the axis of the hub, the rotation of the rollers adding another DOF to the wheel and to the system.

In designing a 3-DOF AGV with omnidirectional wheels, several issues warrant an in-depth study. The orientation of the wheel hubs relative to the platform, the roller orientation with respect to the wheel hub, the number of rollers in a wheel and the roller profiles, are only a few issues that are worth mentioning. Moreover, for an autonomous behavior of the vehicles, it is necessary to have an efficient control algorithm for the on-line computation of the joint parameters. In this paper, the effect of a few kinematics-related parameters on the performance of the vehicles is investigated. A design is then suggested based on the underlying Jacobian matrices that relate the joint rates with the twist of the platform. The twist of any rigid body is understood here as a 6-dimensional vector in the Cartesian space containing the necessary and sufficient information to determine the velocity field throughout the body.

Here, we should emphasize the role of the Jacobian in the control of AGVs, as these systems are *nonholonomic* robots. What this means is that, unlike robotic manipulators where a relation between Cartesian and joint coordinates exists, such a relation exists in AGVs only at the *differential* level, i.e., between Cartesian velocities and joint rates.

2 Jacobian Conditioning

The accuracy of the inverse and direct kinematics results of AGVs depends on the *condition number* of the Jacobian matrices whose inverses are required in calculating the controller setpoints. Moreover, the condition number of a matrix is a measure of the relative roundoff-error amplification of the computed results with respect to the relative roundoff error of the input data, upon solving a system of equations asso-

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ciated with that matrix [4]. Hence, the accuracy of the kinematics results depends on the condition number of the matrices whose inverses are needed. Matrices with small condition numbers produce accurate results, whereas matrices with large condition numbers produce results with correspondingly large roundoff errors. In fact, a condition number equal to unity, which does not introduce any roundoff-error amplification in the solution, is the best that can be achieved. Thus, robustness of the kinematic control is ensured when inverting a matrix with a condition number of unity. Matrices with such a condition number are called *isotropic*.

3 Kinematic Analysis

A general architecture of a 3-DOF AGV, as shown in Fig. 1, is considered for analysis purposes. The vehicle is assumed to consist of λ omnidirectional wheels, μ of which are actuated, and a platform. The platform is coupled by revolute pairs to all the wheel hubs. To obtain a 3-DOF motion of the vehicle, at least three wheels must be actuated, i.e., $\mu \geq 3$.

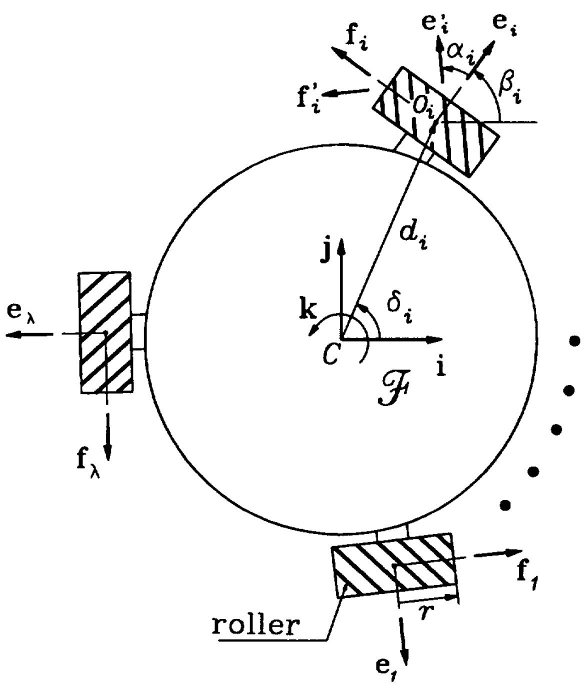

Figure 1: A schematic diagram of a λ-wheeled 3-DOF AGV

The number of rollers in a wheel hub is such that the vehicle moves smoothly. Moreover, only one roller at a time is in contact with the floor. Furthermore, a coordinate frame $\mathcal{F}$ of unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$, with origin at the centroid C of the platform, is attached to this, as shown in Fig. 1. According to this figure, orthogonal unit vectors $\mathbf{e}_i$, $\mathbf{f}_i$ and $\mathbf{e}'_i$, $\mathbf{f}'_i$ $(i = 1, \cdots, \lambda)$ are defined in a horizontal plane. Of these, $\mathbf{e}_i$ and $\mathbf{e}'_i$ are parallel to the axes of the hub and the roller, respectively. Aditionally, $\mathbf{f}_i$ and $\mathbf{f}'_i$ form a right-handed orthogonal triad with $\mathbf{k}$ and $\mathbf{e}_i$ and $\mathbf{k}$ and $\mathbf{e}'_i$, respectively. Note that the roller mounted on the ith wheel and in contact with the floor is termed here the ith active roller. Also, vector $\mathbf{e}_i$ is oriented at an angle β_i with respect to the unit vector $\mathbf{i}$ of the frame $\mathcal{F}$ fixed to the vehicle, as indicated in Fig. 1, and $\mathbf{e}'_i$ is oriented at α_i with respect to $\mathbf{e}_i$, which is parallel to the axis of the hub.

Referring to Fig. 1, the velocity of the centroid of the ith wheel, $\mathbf{v}_i$, can be derived from the velocity $\dot{\mathbf{c}}$ of point C of the platform as

$$\mathbf{v}_i = \dot{\mathbf{c}} + \boldsymbol{\omega} \times \mathbf{d}_i \tag{1}$$

where $\boldsymbol{\omega}$ is the angular velocity of the platform and $\mathbf{d}_i$ is the vector directed from C to O_i. The velocity of O_i can also be determined from the spinning of the rollers and wheel hubs, i.e.,

$$\mathbf{v}_i = -r_r\dot{\theta}'_i\mathbf{f}'_i - r\dot{\theta}_i\mathbf{f}_i \tag{2}$$

where r and r_r are the radii of the wheel hubs and rollers, respectively. The variables $\dot{\theta}_i$ and $\dot{\theta}'_i$ are the joint rates of the ith wheel hub, or wheel, for brevity, and the ith active roller about the axes parallel to $\mathbf{e}_i$ and $\mathbf{e}'_i$, respectively. Now, using eqs.(1) and (2), an equation relating the joint rates of the ith wheel and the ith active roller with the twist of the platform is obtained as

$$\mathbf{J}_i\dot{\boldsymbol{\theta}}_i = \mathbf{K}_i\mathbf{t}_C \tag{3}$$

with the 3×2 matrix $\mathbf{J}_i$ and the 3×6 matrix $\mathbf{K}_i$ given by

$$\mathbf{J}_i = [\,-r\mathbf{f}_i \quad -r_r\mathbf{f}'_i\,], \quad \mathbf{K}_i = [\,-\mathbf{D}_i \quad \mathbf{1}\,] \tag{4}$$

and $\mathbf{D}_i$ is the 3×3 cross-product matrix associated with vector $\mathbf{d}_i$. The 2- and 6-dimensional vectors $\dot{\boldsymbol{\theta}}_i$ and $\mathbf{t}_C$, respectively, are defined as

$$\dot{\boldsymbol{\theta}}_i \equiv [\dot{\theta}_i, \ \ \dot{\theta}'_i]^T, \quad \mathbf{t}_C \equiv [\boldsymbol{\omega}^T, \ \ \dot{\mathbf{c}}^T]^T \tag{5}$$

Note that, for planar motion of the platform, a 3-dimensional twist vector of the platform, defined as $\mathbf{t}'_C \equiv [\dot{\psi}, \ \ \dot{x}, \ \ \dot{y}]^T$, is sufficient to describe the motion of the vehicle. Also, vectors $\mathbf{e}_i$, $\mathbf{e}'_i$, $\mathbf{f}_i$, $\mathbf{f}'_i$, can be expressed in the $\mathcal{F}$ frame of Fig. 1 as

$$\mathbf{e}_i = [c\beta_i,\ \ s\beta_i,\ \ 0]^T, \quad \mathbf{e}'_i = [c\gamma_i,\ \ s\gamma_i,\ \ 0]^T \tag{6}$$

$$\mathbf{f}_i = [-s\beta_i,\ \ c\beta_i,\ \ 0]^T, \quad \mathbf{f}'_i = [-s\gamma_i,\ \ c\gamma_i,\ \ 0]^T \tag{7}$$

where $\gamma_i \equiv \alpha_i + \beta_i$. Upon substitution of eqs.(6 & 7) into eq.(3) and introduction of the 3-dimensional twist vector $\mathbf{t}'_C$ of the platform, an expression relating $\dot{\boldsymbol{\theta}}_i$ with $\mathbf{t}'_C$ is derived, namely,

$$\mathbf{J}_i\dot{\boldsymbol{\theta}}_i = \mathbf{K}_i\mathbf{t}'_C \tag{8}$$

where $\mathbf{J}_i$ and $\mathbf{K}_i$ of eq.(4) are redefined as the 2×2 and the 2×3 matrices displayed below:

$$\mathbf{J}_i = r\begin{bmatrix} s\beta_i & \rho s\gamma_i \\ -c\beta_i & -\rho c\gamma_i \end{bmatrix}, \quad \mathbf{K}_i = \begin{bmatrix} -d_i s\delta_i & 1 & 0 \\ d_i c\delta_i & 0 & 1 \end{bmatrix} \tag{9}$$

with $\rho = r_r/r$ and d_i being the magnitude of the projection of vector $\mathbf{d}_i$ onto the plane of the platform. Furthermore, angle δ_i appearing in eq.(9) is shown in Fig. 1. Vector $\mathbf{d}_i$ can be expressed in the $\mathcal{F}$ frame as

$$\mathbf{d}_i = [d_i c\delta_i,\ \ d_i s\delta_i,\ \ h]^T \tag{10}$$

where h is the distance from the centroid of the ith wheel, point O_i, to the plane of the platform containing point C. It is pointed out here that the parameter h is irrelevant to the kinematics of 3-DOF AGVs.

For the inverse kinematics of the 3-DOF AGVs under study, the actuated joint angles and their time derivatives, which are not independent when redundant actuation is used, are determined from the required twist and twist rate of the platform traversing a desired path. Referring to eq.(8), the 2×2 matrix $\mathbf{J}_i$ is nonsingular, unless angle α_i is equal to 0 or π, because $\det(\mathbf{J}_i) = r^2\rho\ s\alpha_i \neq 0$. Note that a singularity of matrix $\mathbf{J}_i$ occurs at $\alpha_i = 0$ or π, which means kinematically that the wheels become conventional. Thus, for any value of α_i different from those two, $\dot{\boldsymbol{\theta}}_i$ is obtained from eq.(8) as

$$\dot{\boldsymbol{\theta}}_i = \mathbf{J}_i^{-1}\mathbf{K}_i\mathbf{t}'_C \tag{11}$$

where $\mathbf{J}_i^{-1}\mathbf{K}_i$ is given by

$$\mathbf{J}_i^{-1}\mathbf{K}_i = \frac{1}{r\rho\ s\alpha_i}\begin{bmatrix} -d_i\rho\ s(\gamma_i - \delta_i) & -\rho\ c\gamma_i & -\rho\ s\gamma_i \\ d_i\ s(\beta_i - \delta_i) & c\beta_i & s\beta_i \end{bmatrix} \tag{12}$$

From eq.(11), the joint rates of the ith wheel and the ith active roller, $\dot{\theta}_i$ and $\dot{\theta}'_i$, respectively, are given as

$$\dot{\theta}_i = -\frac{1}{r\ s\alpha_i}[d_i\dot{\psi}\ s(\gamma_i - \delta_i) + \dot{x}\ c\gamma_i + \dot{y}\ s\gamma_i] \tag{13}$$

$$\dot{\theta}'_i = \frac{1}{r_r\ s\alpha_i}[d_i\dot{\psi}\ s(\beta_i - \delta_i) + \dot{x}\ c\beta_i + \dot{y}\ s\beta_i] \tag{14}$$

Now, for a λ-wheeled vehicle, two λ-dimensional vectors $\dot{\boldsymbol{\theta}}$ and $\dot{\boldsymbol{\theta}}'$, consisting of all the joint rates of the wheel hubs and the joint rates of the active rollers, respectively, are derived from eq.(11) as

$$\dot{\boldsymbol{\theta}} = \mathbf{L}\mathbf{t}'_C \quad \text{and} \quad \dot{\boldsymbol{\theta}}' = \mathbf{L}'\mathbf{t}'_C \tag{15}$$

where the $\lambda \times 3$ matrices $\mathbf{L}$ and $\mathbf{L}'$ are given as

$$\mathbf{L} = \begin{bmatrix} \eta_1 d_1\ s(\gamma_1 - \delta_1) & \eta_1\ c\gamma_1 & \eta_1\ s\gamma_1 \\ \vdots & \vdots & \vdots \\ \eta_\lambda d_\lambda\ s(\gamma_\lambda - \delta_\lambda) & \eta_\lambda\ c\gamma_\lambda & \eta_\lambda\ s\gamma_\lambda \end{bmatrix} \tag{16}$$

$$\mathbf{L}' = \begin{bmatrix} \eta'_1 d_1\ s(\beta_1 - \delta_1) & \eta'_1 c\beta_1 & \eta'_1 s\beta_1 \\ \vdots & \vdots & \vdots \\ \eta'_\lambda d_\lambda\ s(\beta_\lambda - \delta_\lambda) & \eta'_\lambda & \end{bmatrix} \tag{17}$$

while $\eta_i = -1/(r\ s\alpha_i)$ and $\eta'_i = 1/(r_r\ s\alpha_i)$ for $i = 1, \ldots, \lambda$. In the presence of less actuated wheels than the total number of wheels in the vehicle, i.e., when $\mu < \lambda$, $\boldsymbol{\theta}$ and $\mathbf{L}$ are partitioned as

$$\dot{\boldsymbol{\theta}} \equiv \begin{bmatrix} \dot{\boldsymbol{\theta}}_A \\ \dot{\boldsymbol{\theta}}_N \end{bmatrix} \quad \text{and} \quad \mathbf{L} \equiv \begin{bmatrix} \mathbf{L}_A \\ \mathbf{L}_N \end{bmatrix} \tag{18}$$

Here, $\dot{\boldsymbol{\theta}}_A$ is the μ-dimensional vector consisting of the joint rates of the actuated wheel hubs and $\dot{\boldsymbol{\theta}}_N$ consists of the remaining joint rates, i.e., of the joint rates of the $\lambda - \mu$ non-actuated wheels. Moreover, $\mathbf{L}_A$ and $\mathbf{L}_N$ are the $\mu \times 3$ and $(\lambda - \mu) \times 3$ matrices relating the twist of the platform with the joint rates of the actuated and non-actuated wheels, respectively. Furthermore, a vector of unactuated joint rates $\dot{\boldsymbol{\theta}}_U$ is introduced, which contains all the joint rates of the active rollers for all the wheels and non-actuated wheels. The $(2\lambda - \mu)$-dimensional vector $\dot{\boldsymbol{\theta}}_U$ and the $(2\lambda - \mu) \times 3$ matrix $\mathbf{L}_U$ are defined accordingly, i.e., as

$$\dot{\boldsymbol{\theta}}_U \equiv \begin{bmatrix} \dot{\boldsymbol{\theta}}' \\ \dot{\boldsymbol{\theta}}_N \end{bmatrix} \quad \text{and} \quad \mathbf{L}_U \equiv \begin{bmatrix} \mathbf{L}' \\ \mathbf{L}_N \end{bmatrix} \tag{19}$$

The relations required to obtain the actuated and unactuated joint rates are now readily available from eqs.(15)–(19) as

$$\dot{\boldsymbol{\theta}}_A = \mathbf{L}_A\mathbf{t}'_C \quad \text{and} \quad \dot{\boldsymbol{\theta}}_U = \mathbf{L}_U\mathbf{t}'_C \tag{20}$$

To obtain the twist of the platform from the given actuated joint rates, the relation between the twist of the platform and the actuated joint rates, as given in eq.(20), is used. Since a vehicle with three actuated wheels has three DOF, matrix $\mathbf{L}_A$ is of 3×3. Hence, for a nonsingular $\mathbf{L}_A$ matrix, the twist of the platform is obtained as

$$\mathbf{t}'_C = \mathbf{L}_A^{-1}\dot{\boldsymbol{\theta}}_A = \mathbf{T}_A\dot{\boldsymbol{\theta}}_A \tag{21}$$

where $\mathbf{T}_A \equiv \mathbf{L}_A^{-1}$. For AGVs consisting of more than three actuated wheels, the relation between the twist of the platform and the actuated joint rates leads to more equations than unknowns, i.e., to an overdetermined system of algebraic equations. For a consistent set of input data, contained in vector $\dot{\theta}_A$, vector $\mathbf{t}'_C$ can be calculated with the help of the Moore-Penrose generalized inverse, i.e.,

$$\mathbf{t}'_C = \mathbf{T}_A\dot{\theta}_A \tag{22}$$

where $\mathbf{T}_A \equiv (\mathbf{L}_A^T\mathbf{L}_A)^{-1}\mathbf{L}_A^T$.

The orientation and the position of the point C of the platform in an inertial frame is found by integrating the relation:

$$[\mathbf{t}'_C]_{\mathcal{I}} = [\mathbf{Q}]_{\mathcal{I}}\mathbf{t}'_C \tag{23}$$

where the 3-dimensional vector $[\mathbf{t}'_C]_{\mathcal{I}}$ is the twist of the platform represented in an inertial frame $\mathcal{I}$, whereas the 3×3 matrix $[\mathbf{Q}]_{\mathcal{I}}$ is the orientation of the $\mathcal{F}$ frame with respect to the $\mathcal{I}$-frame. Vector $[\mathbf{t}'_C]_{\mathcal{I}}$ and matrix $[\mathbf{Q}]_{\mathcal{I}}$ are given by

$$[\mathbf{t}'_C]_{\mathcal{I}} \equiv \begin{bmatrix} \dot{\psi} \\ \dot{x}_c \\ \dot{y}_c \end{bmatrix} \text{ and } [\mathbf{Q}]_{\mathcal{I}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\psi & -s\psi \\ 0 & s\psi & c\psi \end{bmatrix} \tag{24}$$

In eq.(24), the variable ψ defines the orientation of the platform, while x_c and y_c define the Cartesian coordinates of point C of the platform in frame $\mathcal{I}$.

4 Design Criteria

It is apparent from eq.(8) that the inversion of matrix $\mathbf{J}_i$ is required in determining the actuated as well as the unactuated joint rates, as in eq.(11). However, matrix $\mathbf{J}_i$ is singular at $\alpha_i = j\pi$, with j being an integer. Moreover, in direct kinematics, where the twist of the platform is calculated from the actuated joint rates, the solution for $\mathbf{t}'_C$, as in eq.(20), is required, which is given by eqs.(21) or (22), i.e., what we need is the evaluation of $\mathbf{L}_A^{-1}$ or $\mathbf{L}_A^I$, where $(\cdot)^I$ stands for the generalized inverse of $(\cdot)$. It is clear from eq.(20), where $\mathbf{L}_A$ is obtained using eq.(16), that if $\gamma_i - \delta_i = j\pi$, or $\gamma_i = j\pi$ or $(2j+1)\pi/2$, for $i = 1,\cdots,\mu$, with j and μ being integers, the latter denoting the number of actuated wheels, then $\mathbf{L}_A$ is rank-deficient. Since $\gamma_i \equiv \alpha_i + \beta_i$ and δ_i are not configuration-dependent, but rather architecture-dependent, the rank-deficiency of $\mathbf{L}_A$ results in a singular design of the vehicle.

The condition number of any $\mu \times k$ matrix $\mathbf{A}$ can be defined as the ratio of the largest to the smallest singular values of $\mathbf{A}$. Now, since the condition number of a matrix measures the roundoff-error amplification upon inversion of that matrix, it is desired to keep the condition number as low as possible. From the foregoing definition, the minimum value that the condition number can attain is unity. This situation occurs when all the singular values of the matrix are identical. Thus, the condition number of the foregoing matrix $\mathbf{A}$ attains its minimum value when the condition below holds:

$$\mathbf{A}^T\mathbf{A} = \sigma^2\mathbf{1} \tag{25}$$

where σ is the k-times repeated singular value of $\mathbf{A}$. Any matrix $\mathbf{A}$ that satisfies eq.(25) is termed *isotropic*. Note that, from eq.(25), the generalized inverse of an isotropic $\mathbf{A}$ is simply $(1/\sigma^2)\mathbf{A}^T$. Equation (25) is the *isotropy condition* for matrix $\mathbf{A}$. This condition will be used presently to attempt isotropic designs of 3-DOF AGVs.

To verify the existence of an isotropic design for inverse kinematics, matrix $\mathbf{J}_i$, given in eq.(9), is written as

$$\mathbf{J}_i = r\begin{bmatrix} s\beta_i & \rho s\gamma_i \\ -c\beta_i & -\rho c\gamma_i \end{bmatrix} \tag{26}$$

where $\gamma_i \equiv \alpha_i + \beta_i$. Angles α_i and β_i and the radius r of the wheel hubs are shown in Fig. 1,while the ratio ρ is given by $\rho = r_r/r$. Now, $\mathbf{J}_i^T\mathbf{J}_i$ is evaluated as

$$\mathbf{J}_i^T\mathbf{J}_i = r^2\begin{bmatrix} 1 & \rho\, c\alpha_i \\ \rho\, c\alpha_i & \rho^2 \end{bmatrix} \tag{27}$$

which is a symmetric and positive-definite 2×2 matrix. It is clear from eq.(27) that, for $\mathbf{J}_i$ to be isotropic, the diagonal entries of matrix $\mathbf{J}_i^T\mathbf{J}_i$ must be identical, whereas its off-diagonal entries must vanish. The latter are zero in turn when $\alpha_i = (2j+1)\pi/2$, for any integer j.

On the other hand, the aforementioned diagonal entries are identical if the ratio $1/\rho^2$ is equal to unity. However, a value of $\rho = 1$ implies that the radius of the wheel hubs is equal to the radius of the rollers, which is not practically feasible. Thus, an isotropic design for inverse kinematics is ruled out.

Since for 3-DOF AGVs with μ actuated wheels the $\mu \times 3$ matrix $\mathbf{L}_A$ of eq.(20) is dimensionally inhomogeneous, the singular values of this matrix have different units, which makes impossible an ordering from smallest to largest, and hence, impossible to calculate the condition number of this matrix. For example, in order to find $\mathbf{L}_A^T\mathbf{L}_A$, the first element of the first column of the matrix product is obtained from the inner product of vector $\mathbf{l}_1$, defined as the first row of matrix $\mathbf{L}_A$, by itself, i.e., $\mathbf{l}_1^T\mathbf{l}_1$. Vector $\mathbf{l}_1$ can be written from eq.(16) as

$$\mathbf{l}_1 \equiv \eta_1[d_1\ s,\xi_1,\ \ c\gamma_1]^T, \tag{28}$$

where η_1 was defined below eqs.(16) and (17), and ξ_1 has been substituted for $\gamma_1 - \delta_1$. The first component of $\mathbf{l}_1$ is dimensionless, whereas the other two components have units of m^{-1}. Thus, the norm of vector $\mathbf{l}_1$ involves the addition of numbers of different dimensions. Apparently, a previous normalization of matrix $\mathbf{L}_A$ is needed, in order to render its entries dimensionally homogeneous. We perform this normalization by dividing the entries of this matrix with units of length by a *characteristic length* L. Moreover, we choose L so that it minimizes the condition number of the dimensionally homogeneous matrix thus resulting. The $\mu \times 3$ matrix $\mathbf{L}_A$ is now redefined as

$$\mathbf{L}_A \equiv \begin{bmatrix} \epsilon_1\ s\xi_1 & \eta_1\ c\gamma_1 & \eta_1\ s\gamma_1 \\ \vdots & \vdots & \vdots \\ \epsilon_\mu\ s\xi_\mu & \eta_\mu\ c\gamma_\mu & \eta_\mu\ s\gamma_\mu \end{bmatrix} \tag{29}$$

where $\epsilon_i = \eta_i\zeta_i$, $\eta_i = -1/(r\ s\alpha_i)$ and $\zeta_i = d_i/L$, for $i = 1,\ldots,\mu$. The left-hand side of eq.(25) with $\mathbf{K}_i$ replaced by $\mathbf{L}_A$, is now derived as the 3×3 symmetric and positive-definite matrix $\mathbf{R}$, namely,

$$\mathbf{R} \equiv \mathbf{L}_A^T\mathbf{L}_A = \begin{bmatrix} R_1 & R_2 & R_3 \\ R_2 & R_4 & R_5 \\ R_3 & R_5 & R_6 \end{bmatrix} \tag{30}$$

where

$$R_1 \equiv \epsilon_1^2\ s^2\xi_1 + \ldots + \epsilon_\mu^2\ s^2\xi_\mu \tag{31}$$
$$R_2 \equiv \epsilon_1\eta_1\ s\xi_1 c\gamma_1 + \ldots + \epsilon_\mu\eta_\mu \tag{32}$$
$$R_3 \equiv \epsilon_1\eta_1\ s\xi_1 s\gamma_1 + \ldots + \epsilon_\mu\eta_\mu \tag{33}$$
$$R_4 \equiv \eta_1^2\ c^2\gamma_1 + \ldots + \eta_\mu^2\ c^2\gamma_\mu \tag{34}$$
$$R_5 \equiv \frac{1}{2}[\eta_1^2\ s2\gamma_1 + \ldots + \eta_\mu^2\ s2\gamma_\mu] \tag{35}$$
$$R_6 \equiv \eta_1^2\ s^2\gamma_1 + \ldots + \eta_\mu^2\ s^2\gamma_\mu \tag{36}$$

Now, from eq.(25), an isotropic design for direct kinematics is achieved if the conditions below are satisfied:

$$R_1 = \sigma^2,\quad R_2 = 0,\quad R_3 = 0,$$
$$R_4 = \sigma^2,\quad R_5 = 0,\quad R_6 = \sigma^2$$

which are the design criteria that we will use below.

5 A Four-Wheeled AGV

Using the criteria derived in Section 4, a 4-wheeled, 3-DOF AGV is designed with all the wheels of the vehicle actuated. Some practical assumptions are made that facilitate the derivations, namely,

$$\delta_1 = \delta,\quad \delta_2 = \frac{\pi}{2} + \delta, \delta_4 = \frac{3\pi}{2} + \delta \tag{37}$$
$$\beta_1 = \beta,\quad \beta_2 = \frac{\pi}{2} + \beta, \beta_4 = \frac{3\pi}{2} + \beta \tag{38}$$

Using eqs.(37) and (38), R_i, for $i = 1,\ldots,6$, of eq.(30) are obtained as

$$R_1 = \epsilon_1^2\ s^2\xi_1 + \epsilon_2^2\ s^2\xi_2 + \epsilon_3^2\ s^2\xi_3 + \epsilon_4^2\ s^2\xi_4 \tag{39}$$
$$R_2 = \epsilon_1\eta_1\ s\xi_1 c\gamma_1 + \epsilon_2\eta_2\ s\xi_2 c\gamma_2 + \epsilon_3\eta_3\ s\xi_3 c\gamma_3 + \epsilon_4\eta_4\ s\xi_4 c\gamma_4 \tag{40}$$
$$R_3 = \epsilon_1\eta_1\ s\xi_1 s\gamma_1 + \epsilon_2\eta_2\ s\xi_2 s\gamma_2 + \epsilon_3\eta_3\ s\xi_3 s\gamma_3 + \epsilon_4\eta_4\ s\xi_4 s\gamma_4 \tag{41}$$
$$R_4 = \eta_1^2\ c^2\gamma_1 + \eta_2^2\ c^2\gamma_2 + \eta_3^2\ c^2\gamma_3 + \eta_4^2\ c^2\gamma_4 \tag{42}$$
$$R_5 = \frac{1}{2}[\eta_1^2\ s2\gamma_1 + \eta_2^2\ s2\gamma_2 + \eta_3^2\ s2\gamma_3 + \eta_4^2\ s2\gamma_4] \tag{43}$$
$$R_6 = \eta_1^2\ s^2\gamma_1 + \eta_2^2\ s^2\gamma_2 + \eta_s^2\ c^2\gamma_3 + \eta_4^2\ s^2\gamma_4 \tag{44}$$

where ϵ_i and η_i, for $i = 1,\ldots,4$, were defined before, as in eq.(29), whereas ξ_i and γ_i of eqs.(39)-(44) are, respectively, $\alpha_i+\beta-\delta$ and $\alpha_i+\beta+(k-1)\pi/2$, for $k = 1,\ldots,4$. Note that, according to our design criteria, $R_2 = R_3 = 0$. Thus, from eqs.(41) and (42), R_2 and R_3 are equal when all conditions below are met:

$$s\gamma_1 = c\gamma_1,\quad s\gamma_2 = c\gamma_2,\quad s\gamma_3 = c\gamma_3,\quad s\gamma_4 = c\gamma_4 \tag{45}$$

In order to satisfy the above conditions we must have

$$\gamma_1 \equiv \alpha_1 + \beta = (2j_1+1)\frac{\pi}{4} \tag{46}$$
$$\gamma_2 \equiv \alpha_2 + \beta + \frac{\pi}{2} = (2j_2+1)\frac{\pi}{4} \tag{47}$$
$$\gamma_3 \equiv \alpha_3 + \beta + \pi = (2j_3+1)\frac{\pi}{4} \tag{48}$$
$$\gamma_4 \equiv \alpha_4 + \beta + \frac{3\pi}{2} = (2j_4+1)\frac{\pi}{4} \tag{49}$$

where j_i, for $i = 1,\ldots,4$, are all integers. Now we show that, if all j_i of eqs.(46)-(49) are set equal to zero, then no isotropic design is possible. However, with $j_1 = j_3 = 0$ and $j_2 = j_4 = 1$, an isotropic design can be achieved. To show this, a set of relations is derived from eqs.(46)-(49) using $\alpha_1 = \alpha$ and $\eta_1 = \eta$, namely,

$$\alpha_2 = \alpha,\quad \alpha_3 = \alpha_4 = \alpha - \pi \tag{50}$$

and

$$\eta_2 = \eta,\quad \eta_3 = \eta_4 = -\eta \tag{51}$$

With the aid of eqs.(50) and (51), the expressions for R_i, for $i = 1, \ldots, 6$, are rewritten, from eqs.(39)–(44), as

$$R_1 = 2\epsilon^2[s^2\xi + s^2(\xi - \pi)] \quad (52)$$

$$R_2 = \epsilon\eta[s\xi c\gamma + s\xi c(\gamma + \frac{\pi}{2}) + s(\xi - \pi)c\gamma + s(\xi - \pi)c(\gamma + \frac{\pi}{2})] \quad (53)$$

$$R_3 = \epsilon\eta[s\xi s\gamma + s\xi s(\gamma + \frac{\pi}{2}) + s(\xi - \pi)s\gamma + s(\xi - \pi)s(\gamma + \frac{\pi}{2})] \quad (54)$$

$$R_4 = 2\eta[c^2\gamma + c^2(\gamma + \frac{\pi}{2})] \quad (55)$$

$$R_5 = \eta^2[s2\gamma + s2(\gamma + \frac{\pi}{2})] \quad (56)$$

$$R_6 = 2\eta^2[s^2\gamma + s^2(\gamma + \frac{\pi}{2})] \quad (57)$$

where $\epsilon = \eta\zeta$, $\eta = -1/(r\ s\alpha)$, $\zeta = d/L$ and $\gamma \equiv \alpha + \beta$. Now, from eqs.(52)–(57), the off-diagonal entries of $\mathbf{L}_A^T\mathbf{L}_A$ for the 4-wheeled vehicles under study vanish. Moreover, the corresponding diagonal entries are equated with σ^2. Then, the following relation is obtained

$$\sigma^2 = \frac{2}{r^2\ s^2\alpha} \quad (58)$$

Again, the isotropy conditions, eq.(37), of the 4-wheeled AGVs under study lead to $\zeta \geq \sqrt{2}/2$ for real values of ξ. Now, with $\zeta = \sqrt{2}/2$ and $\beta = 0$, an isotropic vehicle architecture is obtained, as shown in Fig. 2, its geometrical parameters being given in Table 1. The values for σ and L are calculated as $2/r$ and $\sqrt{2}d$, respectively, with the radius of the wheels r, and d as shown in Fig. 1.

6 Conclusions

Isotropic design criteria for the direct kinematics of a class of 3-DOF AGV were obtained based on the isotropy conditions derived herewith. For a real solution of angle ξ, the said conditions lead to $\zeta \geq \sqrt{2}/2$ for all vehicles under study. If an isotropic design for a 3-DOF AGV is not possible under design specifications, a suitable design may still be achieved by choosing the design variables that minimize the condition number of matrix $\mathbf{L}_A$. Moreover, other design considerations besides kinematics must be implemented in any practical design, optimum designs then being possible upon minimizing a performance index wieghing all figures of merit included, one of which would be kinematic isotropy. Finally, we have extended our isotropic designs to AGVs with more than four omnidirectional wheels.

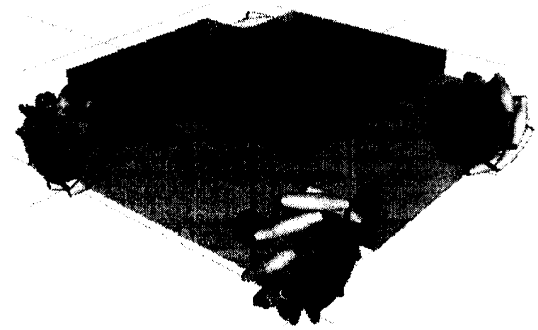

Figure 2: An isotropic 4-wheeled 3-DOF AGV.

Wheel, i	α_i (deg)	β_i (deg)	δ_i (deg)	d_i (m)
1	45	0	−45	d
2	45	90	45	d
3	−135	180	135	d
4	−135	270	225	d

Table 1 Architecture of the AGV of Fig. 2

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