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Modeling and Simulation of Space Robots

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No detail

Abstract

In recent years, research into space robotics have led to the development of several experimental models of space robots that consist of a spacecraft and a manipulator mounted on it. The experimental models accompanied by simulation results are used to study the performances of these robots. However, it is not always feasible to build an experimental model due to space limitation and development cost. Moreover, it is difficult to emulate the three dimensional space environment where experiments can be performed satisfactorily. Thus, it is essential to have a computer simulator with which the space robotics related experiments can ~~be done with much ease. In this paper,~~ a method, based on *the natural orthogonal complement*, ~~introduced elsewhere,~~ is ~~suggested~~ for the development of dynamic models of space robots. As an illustration, the method is used to develop a computer simulator of a space manipulator that moves on a plane. The simulation results can be displayed using an animation program that has been developed in parallel to this research.

Proposed

1 Introduction

In contrast to manipulators on the Earth, whose bases are usually fixed, the base of a space manipulator is mounted on a spacecraft, e.g., a satellite, as shown in Fig. 1. Since a spacecraft is

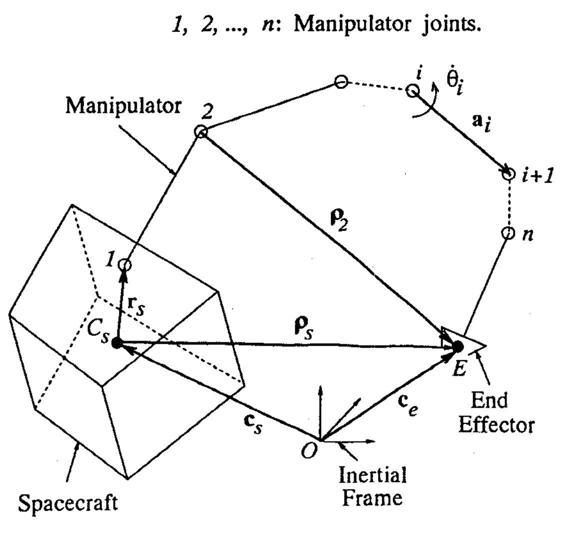

Fig. 1 A schematic diagram of a space robot.

a free moving body its movement is not restricted. Thus, due to dynamic coupling that exists between the spacecraft and the manipulator, spacecraft's configuration changes while the manipulator is in motion. The motion is caused by the reaction forces and moments which are generated at the base of the manipulator. Hence, direct application of the analysis and control schemes for the fixed base robots to space robots is not possible. There are several methods that are proposed for the control of space robots are classified into three categories, as depicted in Fig. 2. In the first category, the position and orientation of the

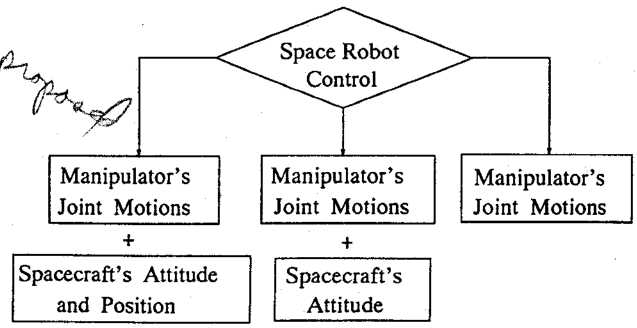

Fig. 2 Control schemes.

spacecraft are kept constant while the manipulator is performing some task like capturing a target or constructing, cleaning or repairing another satellite. This type of requirement arises from the necessity of keeping the on board antennas, which may be required for communication and broadcasting purposes, in a specified position and orientation. The controls of both the spacecraft's position and attitude are, therefore, proposed along with the control of the manipulator joints. Thrusters for position control and Control Moment Gyro (CMG) for attitude control (Komatsu et. al., 1990) may be used for the purpose of controlling the spacecraft. Alternatively, only thrusters (Machida et. al., 1992) or reaction jets (Dubowsky et. al., 1989; Dubowsky and Torres, 1990) can be used to control the spacecraft. Moreover, the manipulator is controlled by its joint actuators. In the second category, an attempt is made to solve the kinematic problem of space manipulators in a way which is similar to that of the fixed base manipulators (Longman et. al., 1987). For that, not the position but the spacecraft's attitude is controlled either by reaction wheels, jets or CMG, whereas the manipulator joint motions are achieved by joint actuators. This method is more complex than the previous one (Papadopoulos and Dubowsky, 1990). However, a method called the *virtual manipulators* (Vafa and Dubowsky, 1987) can simplify the problem. In the third category, only n manipulator joint actuators are used (Umetani and Yoshida, 1989; Koningstein and Ullman, 1989) to control the system at hand. This is possible because the space robot under study has n degrees of freedom (DOF). Note that the space robot containing an n DOF serial manipulator mounted on a spacecraft has $n + 6$ DOF. The additional six DOF is attributed to the spacecraft's ability to move freely in the three

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dimensional Cartesian space. However, in a space environment, under the assumption of no external forces and torques acting on the space robot the total linear and angular momenta of the system are conserved. The momenta conservation contraints are, then, results in six more constraints in addition to the kinematic constraints that exist between the coupled bodies of the system. Hence, n controlling variables, e.g., joint torques, are sufficient to control the space robot where the movement of the spacecraft due to the dynamic coupling is taken into account while evaluating the joint motions. This scheme conserves fuel and electrical power (Papadopoulos and Dubowsky, 1990).

In space robotics, a computer simulator plays an important role, particularly, due to difficulties in realizing the three dimensional space environment. Therefore, most research in space robotics involve computer simulations before space robots are actually built. In this paper, a modeling method based on the scheme mentioned in the third category, i.e., a space robot carrying an n DOF manipulator is controlled by manipulator joint actuators only, is suggested for the development of computer simulators. This scheme is preferred over others because, in dynamics, an independent set of dynamic equations is preferable since the dimension of the problem is minimum, which, in turn, enhances the speed of the simulation algorithm. Moreover, if the set of generalized coordinates is not independent, the solution of a dynamic problem involves the solution of a set of differential and algebraic equations, which is far more complex than the numerical integration of purely differential equations (Gear and Petzold, 1984; Park and Haug, 1986).

In this paper, dynamic model of a space robot that consist of a spacecraft and an n DOF serial manipulator mounted on it is developed using *the natural orthogonal complement* (NOC) (Saha and Angeles, 1991; Saha, 1991). The effectiveness of the method is pointed out and, as an illustration, simulation of a three DOF space robot moving on a plane is reported.

2 Kinematic Modeling

In order to systematically derive kinematic and dynamic equations, some definitions are given as follows: Referring to Fig. 3,

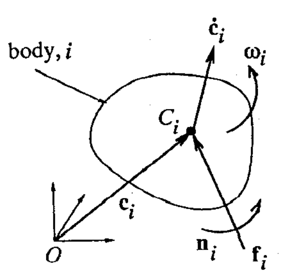

Fig. 3 A free body diagram of a rigid body, i.

$\boldsymbol{\omega}_i$: three dimensional vector representing the angular velocity of the i-th rigid body.

$\dot{\mathbf{c}}_i$: three dimensional vector denoting the velocity of the mass center of the i-th rigid body C_i.

$\mathbf{t}_i$: six dimensional twist vector of the i-th rigid body which is defined as

$$\mathbf{t}_i \equiv \begin{bmatrix} \boldsymbol{\omega}_i \\ \dot{\mathbf{c}}_i \end{bmatrix} \tag{1}$$

$\mathbf{n}_i$: three dimensional vector of applied moment about C_i.

$\mathbf{f}_i$: three dimensional vector of applied force at C_i.

$\mathbf{w}_i$: six dimensional wrench vector of the i-th rigid body which is defined, in parallel with the definition of the twist, as

$$\mathbf{w}_i \equiv \begin{bmatrix} \mathbf{n}_i \\ \mathbf{f}_i \end{bmatrix} \tag{2}$$

In addition,

$\mathbf{I}_i$, m_i: 3×3 inertia matrix of the i-th body about its mass center C_i and the mass of the i-th body, respectively.

$\mathbf{M}_i$: 6×6 *extended mass* matrix of the i-th body which is defined by

$$\mathbf{M}_i \equiv \begin{bmatrix} \mathbf{I}_i & \mathbf{O} \\ \mathbf{O} & m_i\mathbf{1} \end{bmatrix} \tag{3}$$

where $\mathbf{1}$ and $\mathbf{O}$ are the 3×3 identity and zero matrices, respectively. In eq.(3), matrix $m_i\mathbf{1}$ represents the 3×3 matrix whose diagonal elements are m_i and off diagonal terms are all zero.

$\boldsymbol{\alpha}_i$, $\mathbf{h}_i$: three dimensional vectors of angular momentum of the i-th body about C_i and linear momentum of the i-th body, respectively. Vectors $\boldsymbol{\alpha}_i$ and $\mathbf{h}_i$ are given by

$$\boldsymbol{\alpha}_i = \mathbf{I}_i\boldsymbol{\omega}_i \quad \text{and} \quad \mathbf{h}_i = m_i\dot{\mathbf{c}}_i \tag{4}$$

$\mathbf{p}_i$: six dimensional vector of *extended momentum* of the i-th body. This is defined as

$$\mathbf{p}_i \equiv \begin{bmatrix} \boldsymbol{\alpha}_i \\ \mathbf{h}_i \end{bmatrix} \tag{5}$$

Moreover, using the definitions of $\mathbf{t}_i$ and $\mathbf{M}_i$, as in eqs.(1) and (3), respectively, the extended momentum, $\mathbf{p}_i$, of the i-th body is written as $\mathbf{p}_i = \mathbf{M}_i\mathbf{t}_i$. Furthermore, for the system at hand consisting of $n+1$ rigid bodies, a spacecraft and n links of the manipulator, the $6(n+1)$ dimensional vector of the generalized twist and the $6(n+1) \times 6(n+1)$ generalized mass matrix are now defined as follows:

$$\mathbf{t} \equiv [\mathbf{t}_s^T, \mathbf{t}_1^T, \cdots, \mathbf{t}_n^T]^T \quad \text{and} \quad \mathbf{M} \equiv \mathrm{diag}(\mathbf{M}_s, \mathbf{M}_1, \cdots, \mathbf{M}_n) \tag{6}$$

where subscript s stands for spacecraft.

In order to derive kinematic relations for the space robot, first, angular velocity $\boldsymbol{\omega}_e$ and velocity $\dot{\mathbf{c}}_e$ of the end effector are written in terms of the joint variables, i.e.,

$$\boldsymbol{\omega}_e = \boldsymbol{\omega}_s + \dot{\theta}_1\mathbf{e}_1 + \cdots + \dot{\theta}_n\mathbf{e}_n \tag{7}$$

$$\dot{\mathbf{c}}_e = \dot{\mathbf{c}}_s + \boldsymbol{\omega}_s \times \boldsymbol{\rho}_s + \dot{\theta}_1\mathbf{e}_1 \times \boldsymbol{\rho}_1 + \cdots + \dot{\theta}_n\mathbf{e}_n \times \boldsymbol{\rho}_n \tag{8}$$

where $\boldsymbol{\omega}_s$ and $\dot{\mathbf{c}}_s$ represent the angular velocity of the spacecraft and the velocity of its mass center, respectively. Moreover, $\dot{\theta}_i (i = 1, \cdots, n)$ is the joint rate for the i-th revolute pair of the manipulator, whereas $\mathbf{e}_i (i = 1, \cdots, n)$ is the unit vector parallel to the axis of the i-th joint that couples the $(i-1)$st link to the i-th link of the manipulator. Combining eqs.(7) and (8), twist of the manipulator end effector, $\mathbf{t}_e$, is derived as

$$\mathbf{t}_e = \mathbf{J}_s\mathbf{t}_s + \mathbf{J}_m\dot{\boldsymbol{\theta}} \tag{9}$$

where six dimensional vector $\mathbf{t}_s$ is the twist of the spacecraft and n dimensional vector $\dot{\boldsymbol{\theta}}$ contains the joint rates, whereas 6×6 matrix $\mathbf{J}_s$ and $6 \times n$ matrix $\mathbf{J}_m$ are defined as follows:

$$\mathbf{J}_s \equiv \begin{bmatrix} \mathbf{1} & \mathbf{O} \\ -\boldsymbol{\rho}_s \times \mathbf{1} & \mathbf{1} \end{bmatrix} \quad \text{and} \quad \mathbf{J}_m \equiv \begin{bmatrix} \mathbf{e}_1 & \cdots & \mathbf{e}_n \\ \mathbf{e}_1 \times \boldsymbol{\rho}_1 & \cdots & \mathbf{e}_n \times \boldsymbol{\rho}_n \end{bmatrix} \tag{10}$$

Note that the expression $\boldsymbol{\rho}_s \times \mathbf{1}$ in eq.(10) is a 3×3 matrix which can operate on any three dimensional vector $\mathbf{v}$ resulting a new vector $\boldsymbol{\rho}_s \times \mathbf{v}$. Moreover, $\boldsymbol{\rho}_i$ $(i = 1, \cdots, n)$ is the vector denoting the position of the end effector with respect to the i-th joint and $\boldsymbol{\rho}_s$ is equal to $\boldsymbol{\rho}_1 + \mathbf{r}_s$. Vectors $\boldsymbol{\rho}_s$ and $\boldsymbol{\rho}_i$, for $i = 2$, are shown in Fig. 1. Next, since the spacecraft is not controlled the twist of the spacecraft in terms of the joint motions is required. This is done as follows using the principle of momenta conservation: Total angular momentum about the origin of the inertia frame, O, $\boldsymbol{\alpha}$, and linear momentum, $\mathbf{h}$, are written as

$$\begin{aligned} \boldsymbol{\alpha} &= \boldsymbol{\alpha}_s + \mathbf{c}_s \times \mathbf{h}_s + \boldsymbol{\alpha}_1 + (\mathbf{c}_s + \mathbf{s}_{1s}) \times \mathbf{h}_1 + \cdots + \\ &\quad \boldsymbol{\alpha}_n + (\mathbf{c}_s + \mathbf{s}_{ns}) \times \mathbf{h}_n \qquad (11) \\ \mathbf{h} &= \mathbf{h}_s + \mathbf{h}_1 + \cdots + \mathbf{h}_n \qquad (12) \end{aligned}$$

where $\mathbf{s}_{is} \equiv \mathbf{r}_s + \mathbf{s}_{ij}$, for $i = 1, \cdots, n$ and $j = 1, \cdots, i$. Vector $\mathbf{s}_{ij}$ is given by

$$\mathbf{s}_{ij} \equiv \mathbf{a}_j + \mathbf{a}_{j+1} + \cdots + \mathbf{a}_i - \mathbf{r}_i \qquad (13)$$

in which vectors $\mathbf{a}_i$ and $\mathbf{r}_i$ are indicated in Fig. 5. Combining eqs. (11) and (12), vector $\mathbf{p}$ that contains the total angular and linear momenta of the system at hand is expressed as

$$\mathbf{p} = \mathbf{C}_s \mathbf{S} \mathbf{M} \mathbf{t} \qquad (14)$$

where 6×6 matrix $\mathbf{C}_s$ and $6 \times 6(n+1)$ matrix $\mathbf{S}$ are as follows:

$$\mathbf{C}_s \equiv \begin{bmatrix} \mathbf{1} & \mathbf{c}_s \times \mathbf{1} \\ \mathbf{O} & \mathbf{1} \end{bmatrix} \qquad (15)$$

$$\mathbf{S} \equiv \begin{bmatrix} \mathbf{1} & \mathbf{O} & \mathbf{1} & \mathbf{s}_{1s} \times \mathbf{1} & \cdots & \mathbf{1} & \mathbf{s}_{ns} \times \mathbf{1} \\ \mathbf{O} & \mathbf{1} & \mathbf{O} & \mathbf{1} & \cdots & \mathbf{O} & \mathbf{1} \end{bmatrix} \qquad (16)$$

Now, applying the momenta conservation principle, $\mathbf{p} = constant$. Moreover, if the system is at rest before it starts performing any task, i.e., the initial total momenta are zero, then $\mathbf{p} = \mathbf{0}$ at any time during the motion of the robot. Hence, eq.(14) is rewritten as

$$\mathbf{C}_s \mathbf{S} \mathbf{M} \mathbf{t} = \mathbf{0} \qquad (17)$$

The generalized twist, $\mathbf{t}$, whose derivation will be clear in the next section, is then given as

$$\mathbf{t} = \mathbf{T}_s \mathbf{t}_s + \mathbf{T}_m \dot{\boldsymbol{\theta}} \qquad (18)$$

where $\mathbf{T}_s$ and $\mathbf{T}_m$ are $6(n+1) \times 6$ and $6(n+1) \times n$ matrices, respectively. Note that the elements of matrices $\mathbf{S}$ and $\mathbf{T}_s$, as in eqs.(17) and (18), respectively, are such that $\mathbf{S}^T = \mathbf{T}_s$. Therefore, upon substitution of eq.(18) into eq.(17), the following expression is obtained:

$$\mathbf{C}_s(\mathbf{I}_s \mathbf{t}_s + \mathbf{I}_{sm} \dot{\boldsymbol{\theta}}) = \mathbf{0} \qquad (19)$$

where 6×6 symmetric matrix $\mathbf{I}_s$ and $6 \times n$ matrix $\mathbf{I}_{sm}$ are defined as

$$\mathbf{I}_s \equiv \mathbf{T}_s^T \mathbf{M} \mathbf{T}_s \quad \text{and} \quad \mathbf{I}_{sm} \equiv \mathbf{T}_s^T \mathbf{M} \mathbf{T}_m \qquad (20)$$

From eq.(19), vector $\mathbf{t}_s$ is readily obtained as

$$\mathbf{t}_s = -\mathbf{I}_s^{-1} \mathbf{I}_{sm} \dot{\boldsymbol{\theta}} \qquad (21)$$

Equation (21) is the desired expression for $\mathbf{t}_s$ in terms of manipulator joint rates, $\dot{\boldsymbol{\theta}}$.

Finally, upon substitution of eq.(21) into eq.(9) leads to an expression of the twist of the end effector, $\mathbf{t}_e$, which is a linear transformation of vector $\dot{\boldsymbol{\theta}}$, i.e.,

$$\mathbf{t}_e = \mathbf{T}_e \dot{\boldsymbol{\theta}} \qquad (22)$$

where $6 \times n$ matrix $\mathbf{T}_e$ is a function of joint angles and the spacecraft's orientation only, as the spacecraft's position $\mathbf{c}_s$ does not appear in eq.(22). Matrix $\mathbf{T}_e$ is written as

$$\mathbf{T}_e = \mathbf{J}_m - \mathbf{J}_s \mathbf{I}_s^{-1} \mathbf{I}_{sm} \qquad (23)$$

From eq.(23), it is clear that matrix $\mathbf{T}_e$ is the well-known Generalized Jacobian Matrix (Umetani and Yoshida, 1989) of the space robot under study. Moreover, eqs. (22) and (23) are the necessary relations for velocity analyses. For inverse kinematics, where joint variables are calculated for a given set of Cartesian variables of the end effector, joint rates $\dot{\boldsymbol{\theta}}$ are obtained as follows:

1. For $n = 6$, matrix $\mathbf{T}_e$ is a 6×6 matrix. Hence, the inverse of $\mathbf{T}_e$ exists unless the system is in a singular cofiguration. Solution $\dot{\boldsymbol{\theta}}$ is
$$\dot{\boldsymbol{\theta}} = \mathbf{T}_e^{-1} \mathbf{t}_e \qquad (24)$$

2. For $n > 6$, the system is redundantly actuated. Therefore, eq.(22) leads to an underdetermined system of six linear equations in n unknowns that does not define $\dot{\boldsymbol{\theta}}$ uniquely. In this situation, an optimization approach can be undertaken such that the equality constraints of eq.(22) are satisfied. A unique solution, then, be found as
$$\dot{\boldsymbol{\theta}} = \mathbf{T}_e^I \mathbf{t}_e \qquad (25)$$
which minimizes $1/2(\dot{\boldsymbol{\theta}}^T \dot{\boldsymbol{\theta}})$. Matrix $\mathbf{T}_e^I$ is defined as the pseudo-inverse of $\mathbf{T}_e$ (Rao and Mitra, 1971) which is given as
$$\mathbf{T}_e^I \equiv \mathbf{T}_e^T (\mathbf{T}_e \mathbf{T}_e^T)^{-1}$$

It is pointed out here that, contrary to holonomic systems, e.g., a serial or a parallel manipulator, position variables, i.e., joint angles, orientation and position of the spacecraft, are not readily available for given values of orientaion and position of the end-effector since the kinematic constraints of the system under study, eq.(9), are nonholonomic (Nakamura and Mukherjee, 1989). The nonholonomicity of the kinematic constraints implies that no function relating the joint angles of the manipulator with the orientation and position of the end effector exists such that its time derivative is eq.(9). However, if an experimental model of a space robot exists simulation can be carried out using the values of the joint angles that are available from the sensor data. But reliance cannot be made on any sensor data while a computer simulator is attempted because the simulator is expected to predict the behaviour of a space robot even without its real existance. Therefore, the joint angles and the orientation of the spacecraft are obtained by integrating eqs.(24) or (25) and (21), respectively.

For acceleration analysis, eq.(9) is differentiated with respect to time which yields

$$\dot{\mathbf{t}}_e = \mathbf{J}_s \dot{\mathbf{t}}_s + \mathbf{J}_m \ddot{\boldsymbol{\theta}} + \dot{\mathbf{J}}_s \mathbf{t}_s + \dot{\mathbf{J}}_m \dot{\boldsymbol{\theta}} \qquad (26)$$

where $\dot{\mathbf{t}}_s$ is obtained from the time derivative of eq.(19), namely,

$$\dot{\mathbf{C}}_s(\mathbf{I}_s \mathbf{t}_s + \mathbf{I}_{sm} \dot{\boldsymbol{\theta}}) + \mathbf{C}_s(\mathbf{I}_s \dot{\mathbf{t}}_s + \mathbf{I}_{sm} \ddot{\boldsymbol{\theta}} + \dot{\mathbf{I}}_s \mathbf{t}_s + \dot{\mathbf{I}}_{sm} \dot{\boldsymbol{\theta}}) = \mathbf{0} \qquad (27)$$

In eq.(27), $\dot{\mathbf{C}}_s$ is equal to $\boldsymbol{\Omega} \mathbf{C}_s$ where $\boldsymbol{\Omega}$ is a 6×6 matrix that takes the motion of $\mathbf{C}_s$ into account and given by

$$\boldsymbol{\Omega} \equiv \begin{bmatrix} \mathbf{O} & (\boldsymbol{\omega}_s \times \mathbf{c}_s) \times \mathbf{1} \\ \mathbf{O} & \mathbf{O} \end{bmatrix}$$

Moreover, expression $\dot{\mathbf{C}}_s(\mathbf{I}_s \mathbf{t}_s + \mathbf{I}_{sm} \dot{\boldsymbol{\theta}})$ in eq.(27), which is equal to $\boldsymbol{\Omega} \mathbf{C}_s(\mathbf{I}_s \mathbf{t}_s + \mathbf{I}_{sm} \dot{\boldsymbol{\theta}})$, vanishes due to eq.(19). Vector $\dot{\mathbf{t}}_s$ is then solved from eq.(27) and substituted into eq.(26) to obtain the twist rate of the end effector, $\dot{\mathbf{t}}_e$, i.e.,

$$\dot{\mathbf{t}}_e = \mathbf{T}_e\ddot{\theta} + \dot{\mathbf{T}}_e\dot{\theta} \tag{28}$$

where

$$\dot{\mathbf{T}}_e\dot{\theta} \equiv (\dot{\mathbf{J}}_s - \mathbf{J}_s\mathbf{I}_s^{-1}\dot{\mathbf{I}}_s)\mathbf{t}_s + (\dot{\mathbf{J}}_m - \mathbf{J}_s\mathbf{I}_s^{-1}\dot{\mathbf{I}}_{sm})\dot{\theta}$$

3 Dynamic Modeling

Dynamic modeling of a space robot is done using the methodology where a matrix called *the natural orthogonal complement* (NOC) is generated which is orthogonal complement of a kinematic constraint matrix associated to the system under study. The NOC is used to transform the system's uncoupled Newton-Euler equations to an independent set of Euler-Lagrange equations. The said kinematic constraint matrix is obtained by writing existing kinematic constraints between the coupled bodies of the system at hand as linear homogeneous equations in the generalized twist, $\mathbf{t}$. This method was originally proposed for holonomic systems (Angeles and Lee, 1988) and has later been extended to nonholonomic systems (Saha and Angeles, 1991; Saha, 1991). The methodology using the NOC was successfully used for the dynamic modeling of serial manipulators (Angeles and Ma, 1988), parallel robots (Ma and Angeles, 1989), systems with flexible bodies (Cyril et. al., 1989) and Automatic Guided Vehicles (Saha and Angeles, 1989). In this paper, the method is used to model a space robot consisting of a spacecraft and a serial manipulator mounted on it, in order to verify the applicability of the methodology using the NOC to space robotics.

For the purpose of deriving dynamic equations of motion of the sapce robot under study, it is assumed that the space robot consists of $n+1$ rigid bodies, a spacecraft, S, and n rigid links of the manipulator, as shown in Fig. 4. The kinematic constraints

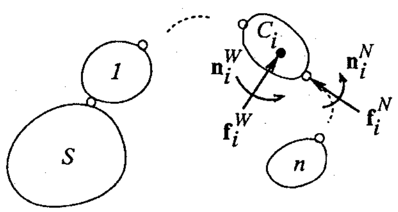

Fig. 4 A serial chain.

are derived in terms of the twists of the individual bodies. This is done as follows: If $\dot{\theta}_j$ represents the joint rate of the j-th revolute pair and $\mathbf{e}_j$ is the unit vector parallel to the axis of rotation of the j-th pair then from Fig. 5

$$\boldsymbol{\omega}_j = \boldsymbol{\omega}_i + \dot{\theta}_j\mathbf{e}_j \tag{29}$$

$$\dot{\mathbf{c}}_j = \dot{\mathbf{c}}_i + \boldsymbol{\omega}_i \times \mathbf{r}_i + \boldsymbol{\omega}_j \times \mathbf{b}_j \tag{30}$$

where, $\boldsymbol{\omega}_i$, $\boldsymbol{\omega}_j$ and $\dot{\mathbf{c}}_i$, $\dot{\mathbf{c}}_j$ are the angular velocities of the i-th and j-th bodies and velocities of the mass centers of the i-th and j-th bodies, respectively, whereas vectors $\mathbf{r}_i$ and $\mathbf{b}_j$ are shown in Fig. 5. Since $\mathbf{e}_j$ is parallel to the relative angular velocity of the j-th body with respect to the i-th body, $\boldsymbol{\omega}_j - \boldsymbol{\omega}_i$, the following holds:

$$\mathbf{e}_j \times (\boldsymbol{\omega}_j - \boldsymbol{\omega}_i) = \mathbf{0} \tag{31}$$

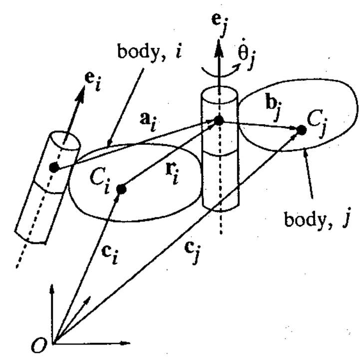

Fig. 5 Two rigid bodies coupled by a revolute pair.

The kinematic constraints in terms of the twists of the i-th and the j-th bodies are then obtained by combining eqs.(30) and (31), namely,

$$\mathbf{A}_i\mathbf{t}_i + \mathbf{A}_j\mathbf{t}_j = \mathbf{0} \tag{32}$$

where 6×6 matrices $\mathbf{A}_i$ and $\mathbf{A}_j$ are the kinematic constraint matrices associated to twists $\mathbf{t}_i$ and $\mathbf{t}_j$, respectively. They are given as follows:

$$\mathbf{A}_i \equiv \begin{bmatrix} -\mathbf{e}_j \times \mathbf{1} & \mathbf{O} \\ \mathbf{r}_i \times \mathbf{1} & -\mathbf{1} \end{bmatrix} \quad \text{and} \quad \mathbf{A}_j \equiv \begin{bmatrix} \mathbf{e}_j \times \mathbf{1} & \mathbf{O} \\ \mathbf{b}_j \times \mathbf{1} & \mathbf{1} \end{bmatrix} \tag{33}$$

Alternatively, the kinematic constraints, eqs.(29) and (30), are expressed in such a way that $\mathbf{t}_j$ is a function of $\mathbf{t}_i$ and θ_j, i.e.,

$$\mathbf{t}_j = \mathbf{T}_i\mathbf{t}_i + \mathbf{T}_j\dot{\theta}_j \tag{34}$$

where $\mathbf{T}_i$ and $\mathbf{T}_j$ are given by

$$\mathbf{T}_i \equiv \begin{bmatrix} \mathbf{1} & \mathbf{O} \\ -(\mathbf{r}_i + \mathbf{s}_{jj}) \times \mathbf{1} & \mathbf{1} \end{bmatrix} \quad \text{and} \quad \mathbf{T}_j \equiv \begin{bmatrix} \mathbf{e}_j \\ \mathbf{e}_j \times \mathbf{s}_{jj} \end{bmatrix} \tag{35}$$

In eq.(35), $\mathbf{s}_{jj}$, defined according to eq.(13), is substituted for $\mathbf{b}_j$. The derivation of eq.(18) is now apparent from eq.(34) since recursive use of the latter for $n+1$ rigid bodies of the system leads to the former. Kinematic relations for other type of joints that may be present in the manipulator in the form of eqs.(32) and (34) can be similarly derived.

The dynamic equations are now derived by writing, first, the Euler equation of rotation of the i-th body with respect to a fixed frame as

$$\mathbf{I}_i\dot{\boldsymbol{\omega}}_i + \boldsymbol{\omega}_i \times \mathbf{I}_i\boldsymbol{\omega}_i = \mathbf{n}_i^W + \mathbf{n}_i^N \tag{36}$$

where $\mathbf{I}_i$ is the inertia matrix of the i-th body about its mass center, whereas $\mathbf{n}_i^W$ and $\mathbf{n}_i^N$ are working external and nonworking constraint moments, respectively, as indicated in Fig. 4. Then, Newton's second law of motion for the i-th body is expressed in the fixed frame as

$$m_i\ddot{\mathbf{c}}_i = \mathbf{f}_i^W + \mathbf{f}_i^N \tag{37}$$

Forces $\mathbf{f}_i^W$ and $\mathbf{f}_i^N$ are defined, similar to $\mathbf{n}_i^W$ and $\mathbf{n}_i^N$, as the working external and nonworking constraint forces acting at the mass center of the i-th body, respectively. Combining eqs.(36) and (37), dynamic equations of motion of the i-th body are written as

$$\mathbf{M}_i\dot{\mathbf{t}}_i + \mathbf{W}_i\mathbf{M}_i\mathbf{t}_i = \mathbf{w}_i^W + \mathbf{w}_i^N \tag{38}$$

where the *extended angular* velocity of the i-th body, $\mathbf{W}_i$, is defined by

$$\mathbf{W}_i \equiv \begin{bmatrix} \boldsymbol{\omega}_i \times \mathbf{1} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{bmatrix}$$

Now, combining eq.(38) for $n+1$ bodies, $6(n+1)$ uncoupled dynamic equations of the system at hand are given as

$$\mathbf{M}\dot{\mathbf{t}} + \mathbf{W}\mathbf{M}\mathbf{t} = \mathbf{w}^W + \mathbf{w}^N \tag{39}$$

where the $6(n+1)$ dimensional generalized twist, $\mathbf{t}$, and the $6(n+1) \times 6(n+1)$ generalized mass matrix, $\mathbf{M}$, are defined in eq.(6), whereas $6(n+1) \times 6(n+1)$ generalized angular velocity matrix $\mathbf{W}$ is given by

$$\mathbf{W} \equiv \text{diag}(\mathbf{W}_s, \mathbf{W}_1, \cdots, \mathbf{W}_n)$$

Moreover, $6(n+1)$ dimensional vectors of working and nonworking wrenches, $\mathbf{w}^W$ and $\mathbf{w}^N$, respectively, are defined in a way similar to the definition of the generalized twist, as given in eq.(6). Using eq.(32), the kinematic constraints that exist between $n+1$ bodies are now obtained as a linear homogeneous equations in the generalized twist as

$$\mathbf{A}\mathbf{t} = \mathbf{0} \tag{40}$$

where $6(n+1)\times 6(n+1)$ matrix $\mathbf{A}$ is the kinematic constraint matrix of the space robot under study. Furthermore, substitution of eq.(21) into eq.(18) leads to an expression of $\mathbf{t}$ which is a linear transformation of the generalized speed, $\dot{\boldsymbol{\theta}}$, i.e.,

$$\mathbf{t} = \mathbf{T}\dot{\boldsymbol{\theta}} \tag{41}$$

For an independent set of joint rates, substitution of eq.(41) into eq.(40) yields

$$\mathbf{A}\mathbf{T} = \mathbf{O} \tag{42}$$

which implies that matrix $\mathbf{T}$ is an orthogonal complement of matrix $\mathbf{A}$. Note that, no special technique is employed, as required in Wehage and Haug (1982), Kamman and Huston (1984) etc. to find orthogonal complement matrices, however, different from $\mathbf{T}$, to evaluate matrix $\mathbf{T}$. Thus, $\mathbf{T}$ is termed as *the natural orthogonal complement* (NOC) of $\mathbf{A}$. In order to find the NOC, if the joint rates cannot be chosen as the set of n independent generalized speeds, as in eq.(41), a different set, for example, twist of the end effector can be chosen to evaluate the NOC which is associated to the twist of the end effector.

Now, multiplication of $\mathbf{T}^T$ to both sides of eq.(39) results in the following dynamical equations of motion of the space robot under study:

$$\mathbf{T}^T(\mathbf{M}\dot{\mathbf{t}} + \mathbf{W}\mathbf{M}\mathbf{t}) = \mathbf{T}^T\mathbf{w}^W \tag{43}$$

where $\mathbf{T}^T\mathbf{w}^N$ vanishes. This can be proved as follows: Power supplied to the system due to $\mathbf{w}^N$, $\mathbf{t}^T\mathbf{w}^N$, is zero, i.e.,

$$\mathbf{t}^T\mathbf{w}^N = 0 \tag{44}$$

Using eq.(41), the foregoing expression is rewritten as

$$\dot{\boldsymbol{\theta}}^T\mathbf{T}^T\mathbf{w}^N = 0 \tag{45}$$

For independent $\dot{\boldsymbol{\theta}}$,

$$\mathbf{T}^T\mathbf{w}^N = \mathbf{0}$$

Finally, using eq.(41) and its time derivative, $\dot{\mathbf{t}} = \mathbf{T}\ddot{\boldsymbol{\theta}} + \dot{\mathbf{T}}\dot{\boldsymbol{\theta}}$, eq.(43) becomes

$$\mathbf{I}\ddot{\boldsymbol{\theta}} + \mathbf{C}\dot{\boldsymbol{\theta}} = \boldsymbol{\tau} \tag{46}$$

where

$\mathbf{I} \equiv \mathbf{T}^T\mathbf{M}\mathbf{T}$: $n \times n$ generalized inertia matrix,

$\mathbf{C} \equiv \mathbf{T}^T(\mathbf{M}\dot{\mathbf{T}} + \mathbf{W}\mathbf{M}\mathbf{T})$: $n \times n$ generalized matrix of convective inertia terms,

$\boldsymbol{\tau} \equiv \mathbf{T}^T\mathbf{w}^W$: n-dimensional vector of generalized external forces.

From the foregoing discussion, then, it becomes apparent that eq.(46) represents the Euler-Lagrange dynamic equations of motion of the system at hand. However, the derivations do not involve lengthy partial differentiations, which would be the case if either a straightforward or a recursive derivation of the Euler-Lagrange equations had been attempted. A point is made here that the derivation of dynamic equations of motion of a space manipulator, compared to its counterpart on the Earth, using the NOC involves writing six additional Newton-Euler equations for the spacecraft, as in eq.(39). Thus, expression of $\mathbf{t}_s$, as in eq.(21), and its time derivative need to be evaluated. Due to the complexity of expression $-\mathbf{I}_s^{-1}\mathbf{I}_{sm}$ the explicit calculation of $\mathbf{t}_s$ from eq.(21) is not recommended. It is efficiently evaluated using the scheme to find the NOC of a complex system (Saha and Angeles, 1991) and noting that matrix $\mathbf{I}_s$ is a symmetric positive deinite matrix whose Cholesky decomposition exists (Stewart, 1973). Additionally, when a set of variables, different from the joint rates, is chosen as the independent set of generalized speed the control variables, e.g., joint torques, cannot be calculated directly from the dynamic equations of motion, eq.(46), because the generalized force vector, $\boldsymbol{\tau}$, does not contain the joint torques. Thus, an additional scheme is required. One such scheme is suggested in Saha (1991) where the power supplied to the system is expressed both in terms of the joint variables and the independent set of variables—twist of the end effetor can be a choice of the independent set—which are then equated. The solution of the resulting underdetermined system gives the values for the desired control variables.

4 An Example: A Space Manipulator Moving on a Plane

It is assumed that a space robot that consists of a three-link manipulator mounted on a spacrcraft, as shown in Fig. 6, is moving on a plane. The system has three DOF and is controlled by only three joint actuators. For smooth motion of the robot, trajectory planning is done such that the robot starts and stops with zero velocity and acceleration. For simulation, first, the position and orientation of the end effector, E, and their first and second derivatives are given as inputs to find the required joint torques. This step is called inverse dynamics. These calculated joint torques, as controlling commands, are then used for simulation purposes.

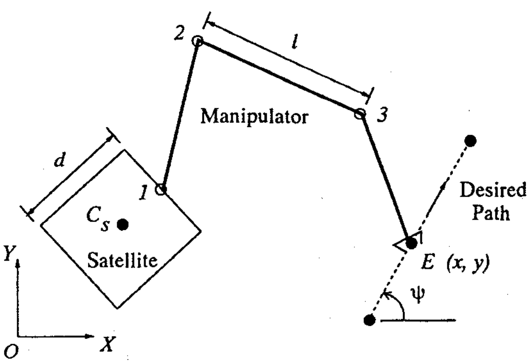

Fig. 6 A space robot moving on a plane.

4.1 Trajectory Planning

The space robot for which the results are reported here has three degrees of freedom. Thus, angle ψ, and coordinates x and y, as shown in Fig. 6, are sufficient to specify orientation and position of the end effector, respectively. In order to start and stop with zero velocity and acceleration, variables ψ, x and y are assumed to be 5th-order polynomial functions of time. If f_j represents either ψ or x or y then the polynomial function is given by

$$f_j = a_{0j} + a_{1j}t + \cdots + a_{5j}t^5 \tag{47}$$

where coefficients a_{ij}, for $i = 0, \cdots, 5$, are to be determined from the boundary conditions given below:

$$\text{at} \quad t = t_1; \qquad f_j = f_{1j}, \quad \frac{df_j}{dt} = \frac{d^2 f_j}{dt^2} = 0 \tag{48}$$

$$\text{and at} \quad t = t_2; \qquad f_j = f_{2j}, \quad \frac{df_j}{dt} = \frac{d^2 f_j}{dt^2} = 0 \tag{49}$$

Variables ψ, x and y are then calculated with $t_1 = 0$, $\psi_1 = -1.385$ rad, $x_1 = 1.4625$ m, $y_1 = 0.2906$ m, and $t_2 = 120$ sec, $\psi_2 = 0$, $x_2 = 1.951$ m, $y_2 = 0.875$ m. Variations of ψ, x and y vs. time are shown in Fig. 7 for step size $\Delta t = 2$ sec. In Fig. 7, "psi", "x" and "y" represent variables ψ, x and y, respectively.

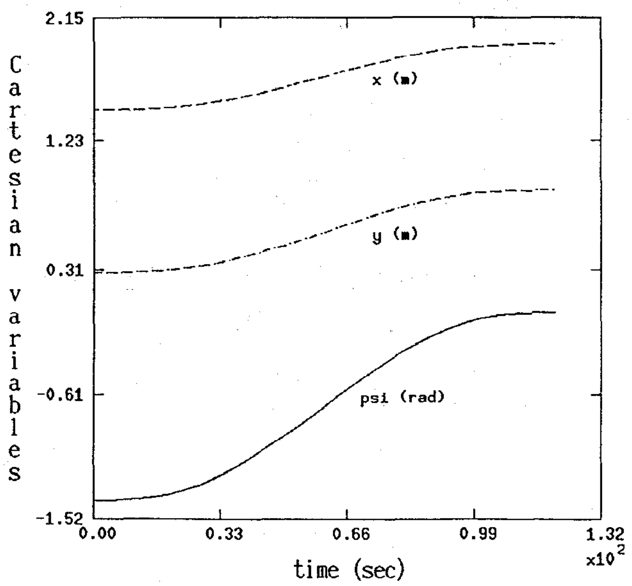

Fig. 7 Desired path: variations of ψ, x and y vs. time.

4.2 Inverse Dynamics

Inverse dynamics results for the space robot moving on a plane are obtained without any consideration of dissipation and flexibility in the links or couplings. The essential dimensions and the inertial parameters of the space robot under study are given as: Referring to Fig. 6,

For the spacecraft,
width, $d = 0.5$ m, mass $= 50$ kg
For each manipulator link (all links are identical),
length, $l = 0.3$ m, diameter $= 0.05$ m, and mass $= 6$ kg

Now, for desired values of ψ, x, y, and their first and second derivatives joint torques are calculated from the dynamic equations of motion, eq.(46). Fig. 8 shows the variations of the joint torques, "tau_1", "tau_2" and "tau_3" that act on joints 1, 2 and 3 of the manipulator, respectively, vs. time.

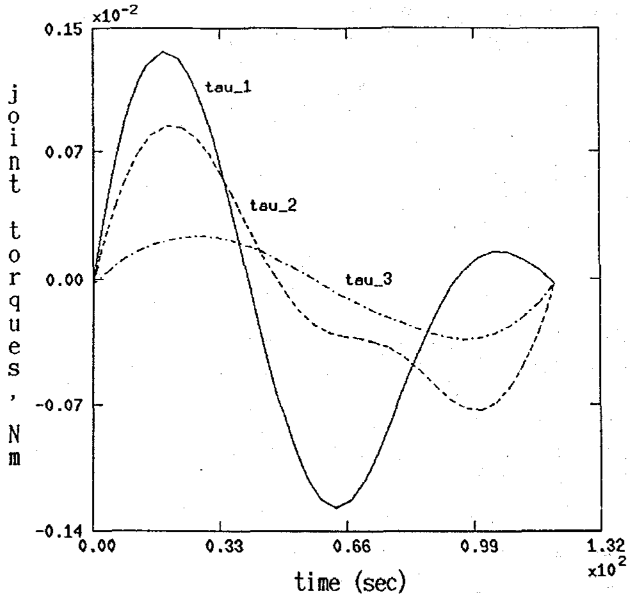

Fig. 8 Joint torques required to follow the desired path.

4.3 Simulation and Animation

The torques that are obtained in inverse dynamics are now supplied to the simulation program as the controlling torques of the space manipulator. Simulation results are obtained by integrating the dynamical equations of motion, eq.(46), using Runge-Kutta-Fehlberg method (Butcher, 1987). The deviation of the simulated path from the desired path is shown in Fig. 9. The simulation errors—an error is calculated as the difference between the desired and the simulated values of the variable under investigation—are found to be quite small. The maximum error, about 2%, is found in the values of coordinates along X-axis. The small simulation errors are attributed mainly to the accuracy of the developed algorithm for the system at hand. Apparently, the major sources of errors of the simulation model lie in the solutions of $\mathbf{t}_s$ and $\ddot{\boldsymbol{\theta}}$ that require the inversion of matrices $\mathbf{I}_s$ of eq.(21) and $\mathbf{I}$ of eq.(46), respectively. Since, both

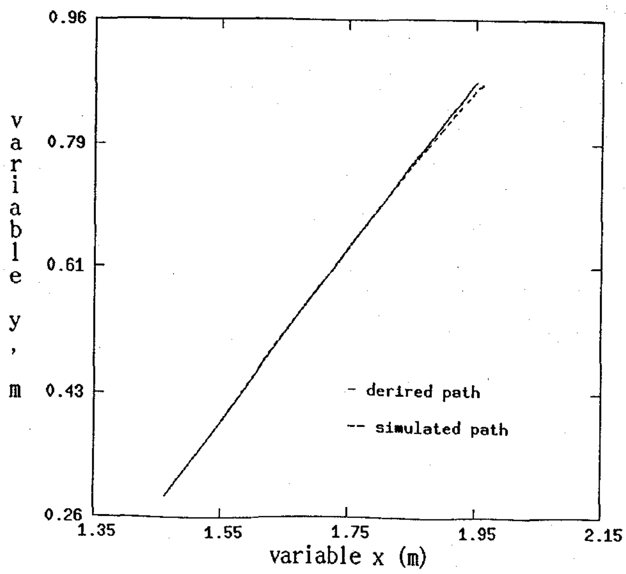

Fig. 9 Desired and the simulated paths in X-Y plane.

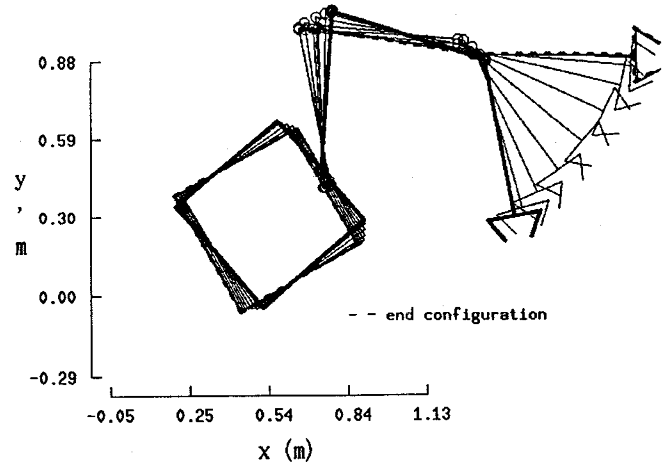

Fig. 10 Intermediate configurations during animation.

matrices, $\mathbf{I}_s$ and $\mathbf{I}$, are symmetric positive definite Cholesky decomposition which is *unconditionally* stable algorithm (Stewart, 1973) is used to solve for $\mathbf{t}_s$ and $\ddot{\theta}$. However, inaccuracies in the symmetric positive definiteness of matrices $\mathbf{I}_s$ and $\mathbf{I}$ depend on the choice of algorithm used to generate them. The proposed dynamic modeling using the NOC does not involve any scheme which accumulates large errors either in generating matrices $\mathbf{I}_s$ and $\mathbf{I}$ or any other system variables of the dynamic model, as in eq.(46).

An animation software based on the *X-library* is developed in SPARC LT AS1000/E20 that allows one to see the motion of the space manipulator. Figure 10 shows some intermediate configurations of the space robot while an animation is performed.

5 Conclusions

A methodology based on *the natural orthogonal complement* (NOC) is developed for the simulation of space robots where a serial manipulator is mounted on a spacecraft. The methodology is, however, applicable to parallel systems as well, i.e., several serial manipulators are mounted on a spacecraft. Moreover, an outline is given for systems with redundant actuations. Simulation and animation software are developed for a three degrees of freedom planar space robot. The results show very little error, thus, confirming the stability of the developed simulation scheme.

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