Synthesis and Analysis of a New Mechanism for Sheep Shearing Machine

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Abstract

A sheep shearing machine is extensively used by the wool industries to cut hair from the sheep body. In this paper, synthesis of a new mechanism for this machine is described. A number of ideas are considered and are compared using a 4-point scale with respect to different critical aspects like cost, weight, wear, etc. to yield a four-bar spatial spherical mechanism. Next, the dimensional synthesis of the selected mechanism is done, and the safe minimum dimensions are obtained by applying the strength criteria against given input torque and speed. After this the proposed mechanism is analyzed kinematically using the vector approach. Further, a Matlab code is written to get the variation of the cutter displacement against the crank rotation. The resulting mechanism shows promise to replace the existing mechanism with several advantages.

Keywords: Sheep, Shearing, Mechanisms, Synthesis, Analysis

1 Introduction

A sheep shearing machine used by the wool industries [1] is mainly consisting of a spatial four-bar mechanism [2] driven by an electric motor (Fig. 1). Rotational input is provided to the crankshaft of the machine by a flexible coupling connected to an electric motor to obtain the output-rocking motion of the fork and the cutter, relative to the comb. This relative motion causes the shearing action of the hair as the handheld machine is moved over the body of the sheep like the hair cutting in a barber shop. There is also an arrangement to vary the pressure on the cutter by turning a nut. Important features in this sheep shearing machine are the displacement of the cutter relative to the comb (cutting length), the pressure required to maintain necessary clearances between the cutter and comb, and the power consumption. This apparatus of the Sheep Shearing Machine is not currently manufactured in India, but needs to be imported. The cost involved in this transaction is huge. Keeping the cost consideration in mind some mechanisms that can be used in place of existing one are proposed here and are compared in a way to get the one that suits the requirement in most efficient way.

The paper is organized as follows: Section 2 presents the selection of the new mechanism, followed by the dimensional synthesis in section 3. Section 4 presents the kinematic analysis. Finally, results and conclusions are provided in the sections 5 and 6, respectively.
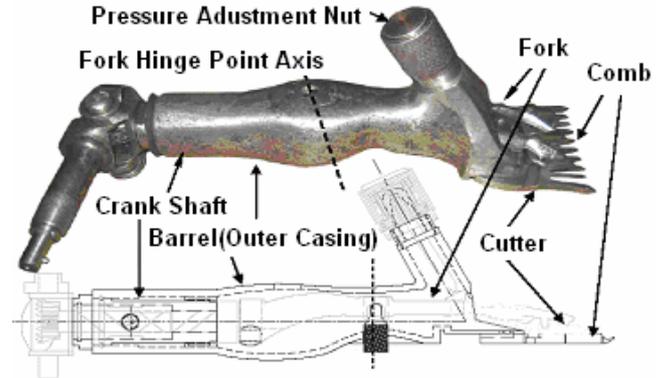
Figure 1: An Existing Sheep Shearing Machine

2 Selection of New Mechanism

In this section, the main concern is on different working mechanisms that are cost effective, compact, and can fulfill the torque requirement, the cutting length, and the life criteria.

2.1 Planar Mechanisms

All the links of such mechanisms lie in a single plane and thus each unconstrained link can have a maximum of three degrees of freedom.

2.1.1 Crank-Rocker 4-Bar Mechanism (Idea P1)

Rocker as the Cutter: Here, power is given to link O_AA and link O_BB act as cutter, whereas the comb is part of the frame (Fig. 2).
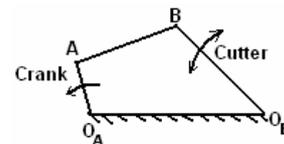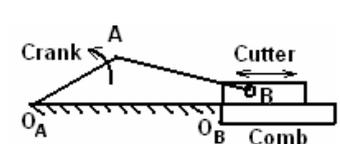
Figure 2: Idea P1   **Figure 3: Idea P2**

2.1.2 Slider-Crank 4-Bar Mechanism (Idea P2)

Slider as the Cutter with teeth on its front: Here, either teeth on the cutter and comb are inclined (unlike straight teeth in current apparatus), or the axis of cutter's motion is inclined with respect to the comb (Fig. 3).

2.1.3 Slider-Crank 4-Bar Mechanism (Idea P3)

Slider as the Cutter with teeth on its Side (Fig. 4): Here, translation of the slider over the comb does the cutting action.
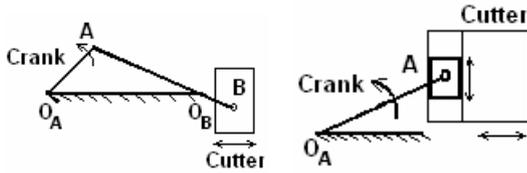
Figure 4: Idea P3   **Figure 5: Idea P4**

2.1.4 Double Slider Mechanism (Idea P4)

Second Slider as the Cutter: Its working is similar to idea P3 but more compact (Fig. 5).

2.1.5 Quick-Return Mechanism (Idea P5)

Slotted Lever as the Cutter: Here, the advantage of quick return stroke of this mechanism is utilized during cutting process. But for this, the cutter has to be hinged at a larger distance (Fig. 6).
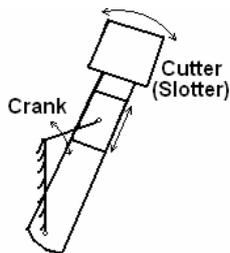
Figure 6: Idea P5

2.1.6   Six-Bar Mechanism (Idea P6)

Links 4 and 5 are connected to the four-bar linkage ABCD with links 1, 2 and 3. Point F of two-bar group EFG, connected to rod 2 and rocker arm 3, describes a complex path. A cutter at this point can be utilized for the cutting purpose (appears from the traced curve shown in Fig. 7).
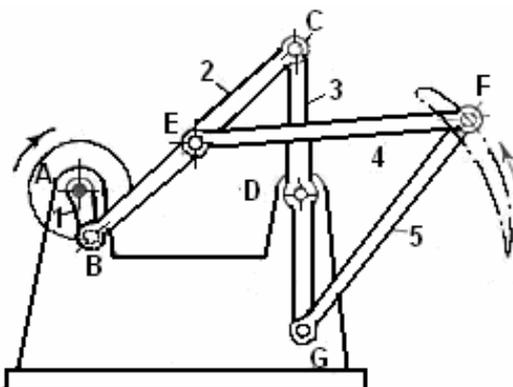
Figure 7: Idea P6 [3]

2.1.7 Six-Bar Mechanism with Adjustable Driven Link Oscillation (Idea P7)

Upon rotation of crank 1, driven link 2 oscillates. The angle of oscillation of link 2 can be varied by changing the position of hinge A by means of screw device 3 which moves slide block 4 along fixed guides a-a. The oscillation of link 2 can be utilized for the cutting action (Fig. 8).
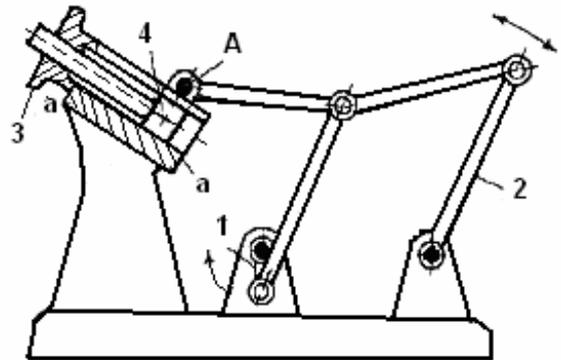
Figure 8: Idea P7 [3]

2.2   Spatial Mechanisms

The links of such mechanisms are not bound to move in a plane and hence each unconstrained link can have six degrees of freedom.

2.2.1   Four-Bar Spherical Mechanism (Idea S1)

As shown in Fig. 9, axes a, b, c and d of all the kinematic turning pairs of the mechanism intersect at a single common point O. Axes a and d are perpendicular. Axes a and b are at an angle of 45^0. Upon rotation of crank 1, link 2 oscillates with amplitude of 90^0. This oscillation of link 2 can be utilized for obtaining the desired cutting action.
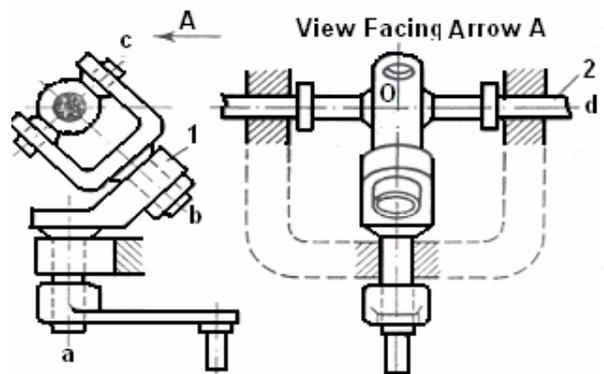
Figure 9: Idea S1 [3]

2.2.2   Four-Bar Spherical Mechanism (Idea S2)

Crank 1 (Fig. 10) rotates about fixed axis z-z. Link 2 is connected by turning pairs to crank 1 and link 3 which turn about fixed axis y-y. In rotation of crank 1 about axis z-z, link 3 oscillates about axis y-y under the condition that the

axes of all the kinematic turning pairs intersect at a single point. This oscillation of link 3 can be utilized for obtaining the desired cutting action.
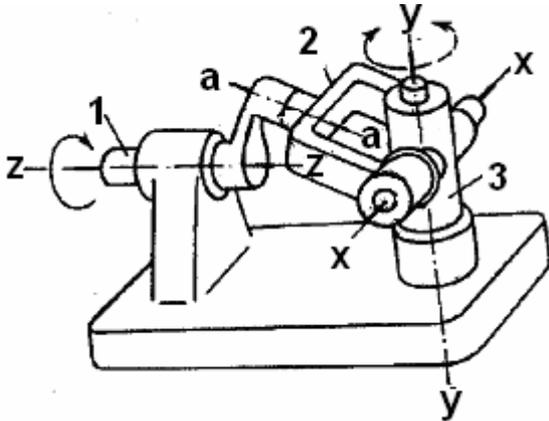
Figure 10: Idea S2 [3]

2.3 Comparison

Five criteria (Table 1) are used to evaluate the above mechanisms and these criteria are assigned weights as per there importance. A 4-point scale, as reported in [4], is shown in Table 2. Note that the numbers 1, 4, 7 and 10 are chosen so that there is sufficient margin between any two values, as the ideas are compared qualitatively. One could, however, choose a different set like 0, 3, 6, 10, or even the ranges like 0-2, 3-4, 5-7, 8-10, for the same purpose. For comparison, each criterion of every idea is checked against the 4-point scale and the summation of the product of marks and weights gives the total score for an idea (Table 3). For example, cost is considered here as the most important criterion. Hence, it is given maximum weightage. Further, if none of the ideas ideally suits the cost criterion no idea gets 10 marks, as indicated in Table 2. Assume, however, that idea P1 suits it well, and, therefore, its score is 70, i.e., 10×7, for the cost criterion. Similarly other criteria are evaluated and put in the column P1 of Table 3. For other ideas, scores are similarly calculated and tabulated in Table 3. Finally, total scores are calculated for all the ideas.

Table 1: Criteria

Criteria	Weights
Cost	10
Weight	7
Size	7
Wear/Flexibility	7/4
Existing Component Use	4

Table 2: 4-Point Scale

Condition	Marks
Ideally Suited or Exceptionally Important	10
Well Suited or Important	7
Average Suitability	4
Insignificant	1

Table 3: Total Score for Various Ideas

Criteria	Idea								
	P1	P2	P3	P4	P5	P5	P6	S1	S2
Cost	70	70	70	40	70	40	10	70	**70**
Weight	70	70	70	70	70	49	28	70	**70**
Size	28	70	28	28	28	49	28	49	**70**
Wear/Flexibility	70	28	28	28	49	16	28	16	**16**
Existing Component Use	28	16	28	16	16	16	16	28	**28**
Total Score	266	254	224	182	233	170	110	233	**254**

Note that ideas P1, P2 and S2 have got comparable scores. The planar mechanisms have the obvious advantage of simplicity when compared with complex spatial one. However, these mechanisms seem to be less feasible for the present application due to

1. the limitation of single plane in planar mechanisms results into larger area
2. the larger area in case of planar mechanism also affects the holding comfort.
3. note able to use a number of components from the existing mechanism like crank, comb-cutter and even pressure pin.

Therefore, the Four-Bar Spherical Mechanism (Idea S2) is chosen as an alternative mechanism for the sheep shearing machine under consideration

2.4 Selected Mechanism

To use the chosen mechanism as an alternative to the existing cutting apparatus, certain modifications are required. These are

1. Cutter has to be attached to link #3 (Fig. 11).
2. Link #1 has to have a disc to hold the coupler (link #2) and forming a spherical joint.
3. Axes a-a, x-x, y-y and z-z will intersect at one common point (O).
4. Axis a-a is inclined with z-z at an angle φ, such that the coupler (link #2) will generate a cone of half angle φ while in motion.
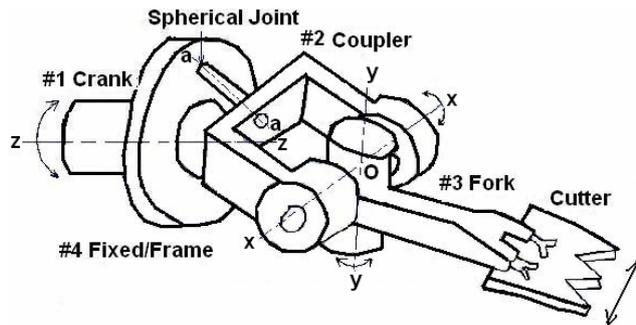
Figure 11: Idea S2-based Sheep Shearing Machine

2.5 Working

Link #1 is attached with the motor to have the rotary input. Links #1 and #2 have relative motion along axis a-a which is inclined, this inclination helps link #2 to have turning motion about axis a-a, x-x and y-y. Motion of link #2 about y-y is actually due to revolute joint between link #3 and #4 (fixed). Hence, the motion of link #3 generates the desired oscillatory motion of the cutter.

3 Dimensional Synthesis

For this, the geometrical and the strength consideration are taken into account.

3.1 Geometric Consideration

In this, geometrical relationships between different links have been established to put limit on the link lengths. Figure 12 shows the different views of the mechanism for the two extreme positions of the crank so that it does not hit the disk or crank while in operation.
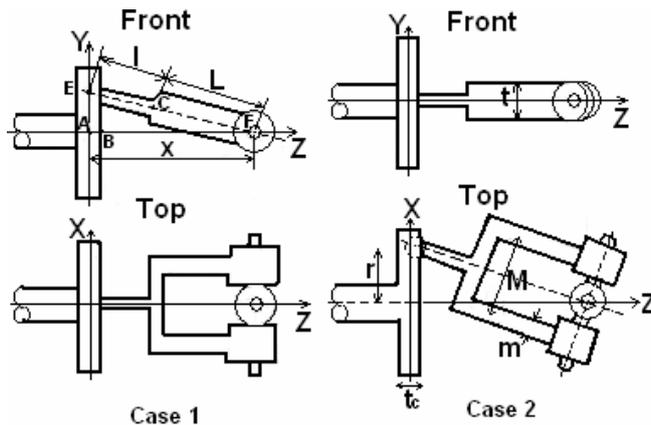
Figure 12: Top and Front Views of the Mechanism

Case 1: When the crank is at the starting point, i.e., angle $\theta = 0^0$. Figure 13 shows the position of the links.

Case 2: When the crank is at an angle $\theta = 90^0$. In this case the coupler can interfere with the crank only when the length BC is perpendicular with the length EF (Fig. 13).
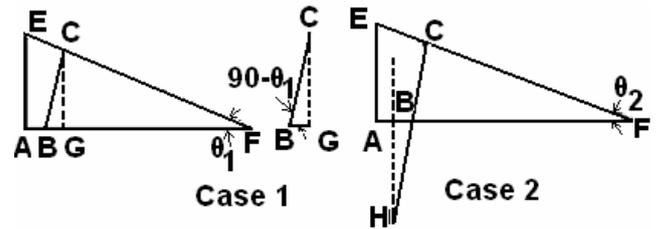
Figure 13: Line Diagram of the Mechanism

From above two cases, it is observed that the angle AFE is half cone angle made by the coupler axis, i.e, $\theta_1 = \theta_2 = \varphi_{max}$, where φ is the orientation angle of fork/cutter with Z-axis and φ_{max} is its maximum value. Therefore,

$$AF = x = \frac{r}{\tan\varphi_{max}} = \frac{t_c}{2} + \frac{t}{2}[Sin\varphi_{max} + \frac{Cos\varphi_{max}}{\tan\varphi_{max}}];$$

$$t = \frac{2*CG}{Cos\varphi_{max}}, L = \frac{CG}{Sin\varphi_{max}};$$

and

$$EF = l + L = \frac{r}{Sin\varphi_{max}}$$

To determine these dimensions, some data are taken from the existing mechanism, i.e.,

Crank radius, r = 8 mm, and

Half-cone angel, $\varphi_{max} = 7.5946^0$

Therefore,

x = 60 mm, and EF = 60.53 mm

Now, if t_c is assumed to be 8 mm, i.e., t = 8 mm, then

t = 14 mm, L = 50 mm, and l = 10.53 mm

From the knowledge of all these dimensions, and Fig. 12 (Case 2), finally,

M = 2(CH) = 40 mm.

3.2 Strength Consideration

For the complete dimensional synthesis of the mechanism, each link is now considered separately and subjected to strength criteria [5].

1. **Input Shaft and the Disc (Fig. 14):** Input shaft is subjected to torque T. The value is taken from the existing machine which is, T = 0.8526 Nm. Therefore, it should be designed in torsion and checked in fatigue due to fluctuating force F. For l_s = 20 mm with SAE C-20 hot rolled steel (Note: Other materials were also considered), the application of Von-Mises shear stress theory, yields a safety factor of 2.2 in fatigue when the shaft diameter, d_s = 8 mm, and disc diameter D = 28 mm
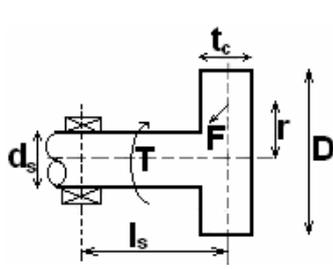
Figure 14: Shaft and Disc   **Figure 15: Coupler**
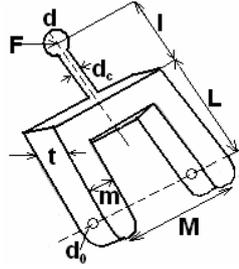
2. **Coupler (Fig. 15):** It is connected to the crank which is actually the disk in Fig. 14 with spherical joint and to the fork with a revolute joint. It is also subjected to the fluctuating load and forms a cone of semi-angle φ_{max} during rotation. Following the static design with fatigue check in bending for C-20 steel, we get, $d_c = 6$ mm (Safe life: $3.8*10^8$ cycles).

3. **Coupler Modification (Fig. 16):** To reduce the weight of the mechanism, coupler arms are modified as in Figure 17. Design checks at four critical sections A, B, C and D leads to $m' = 6$ mm and $t' = 10$ mm ensuring factor of safety in fatigue greater than 5.

4. **Fork (Fig. 17):** It has a tubular cross-section and is attached with the frame and to the coupler through revolute joints. The two protruding arms are subjected to fluctuating reactions from the coupler. These reactions involve the component of bending as well as compression. Again starting with static design and then checking for fatigue (C-20) yields, $d_o = 6$ mm (Safety factor: 3.3). Further, $t = 14$ mm and $m = 10$ mm, yields a safety factor of 2.3 at the coupler ends in crushing, which is a conservative design. For compact size,

$$D_1 = M - m = 40 - 10 = 30 \text{ mm, and}$$

$$D_2 = 0.8*D_1 = 24 \text{ mm.}$$
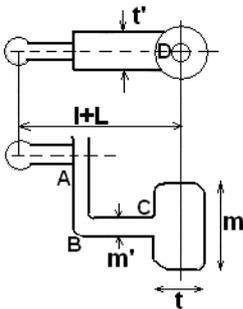
Figure 16: Modified Coupler   **Figure 17: Fork**
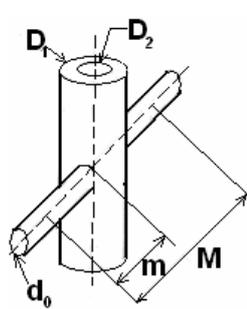
4 Kinematic Analysis

Once the synthesis of the new mechanism for the sheep shearing machine at hand is complete, its kinematic analysis is performed in this section to check the performance of the machine. Since the mechanism has one-degree-of-freedom one motor input is used to derive

the mechanism. For analysis of mechanism, it is represented in the vector form (Fig. 18) and a closed loop equation is formed. Let,

AF = x: vector representing the fixed link.

AE = r: vector representing the crank (XY plane) making angle θ with X-axis.

EI = q: vector representing the coupler link making angle α, β and γ with X, Y and Z axes respectively. As indicated in the Figure 18, point I is chosen such that it lies on the intersection of the line perpendicular to the vectors, **x-r** and the axis a-a (Fig. 11)

IF = p: vector representing the output link (fork) in XZ plane making angle φ with Z-axis.
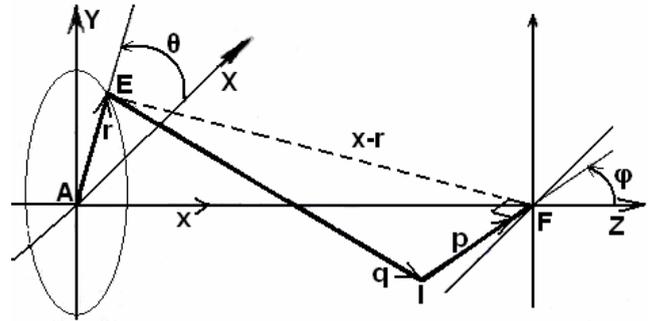
Figure 18: Vector Diagram

The governing closed loop equation is then given by [6]

$$\mathbf{p} + \mathbf{q} + \mathbf{r} = \mathbf{x} \tag{1}$$

Resolving these vectors into their components gives

$$\begin{Bmatrix} p\mathrm{Sin}\varphi \\ 0 \\ p\mathrm{Cos}\varphi \end{Bmatrix} + \begin{Bmatrix} q\mathrm{Cos}\alpha \\ q\mathrm{Cos}\beta \\ q\mathrm{Cos}\gamma \end{Bmatrix} + \begin{Bmatrix} r\mathrm{Cos}\theta \\ r\mathrm{Sin}\theta \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ x \end{Bmatrix} \tag{2}$$

Also, as vector **q** make angles α, β and γ with X, Y and Z axes, therefore

$$\mathrm{Cos}^2\alpha + \mathrm{Cos}^2\beta + \mathrm{Cos}^2\gamma = 1 \tag{3}$$

4.1 Displacement Analysis

The disc, i.e., the crank is attached with the motor and the motion of fork/cutter is of interest. Therefore, to express the output angular displacement in terms of the input angular displacement Eq. (2) is arranged as

$$q\mathrm{Cos}\alpha = -(r\mathrm{Cos}\theta + p\mathrm{Sin}\varphi) \tag{4}$$

$$\mathrm{Cos}\beta = \left[\frac{-r\mathrm{Sin}\theta}{q} \right] \tag{5}$$

$$q\mathrm{Cos}\gamma = x - p\mathrm{Cos}\varphi \tag{6}$$

Squaring and adding Eqs. (4) and (6) and using Eqs. (3) and (4), we can express the output angle, φ, in terms of input angle, θ, as

$$(c-a)t^2 + 2bt - (a+c) = 0$$

where

$$a=\frac{(q^2\text{-}r^2\text{-}p^2\text{-}x^2)}{2},$$

$$b=prCos\theta, \quad c=xp \quad and \quad t=tan\frac{\varphi}{2}$$

Further, from the vector diagram it can be observed that

1. Magnitude of vector **q** can be obtained from the right angle triangle, EFI, whereas the magnitude of the vector, **x-r**, can be obtained from another right-angle triangle, FAE. Hence, $q^2=r^2+p^2+x^2$

2. Vector **x** and **r** are always perpendicular to each other

3. The resultant of vectors **x** and **r** for all values of θ will be perpendicular to vector **p**.

Due the result obtained in item 1 above, 'a' of the quadratic equation after Eq. (6) will vanish, i.e., a = 0. Accordingly, the solutions are

$$t_{1,2}=\frac{\text{-}b\pm\sqrt{b^2+c^2}}{c} \tag{7}$$

Since $\varphi_{1,2}=2tan^{-1}t_{1,2}$, the output angular displacement depends only on input motion and the link lengths. From Eq. (7), for a single input, we get two outputs. This observation can be explained by the fact that for any given value of θ the four-link mechanism can be formed in two ways.

4.2 Velocity Analysis

The crank is given constant angular speed but the fork/cutter will have varying angular speed. To get the relationship between the output and the input angular velocities, Eqs. (2) and (3) are differentiated with respect to time, i.e.,

$$\begin{Bmatrix} p\dot{\varphi}Cos\varphi \\ 0 \\ \text{-}p\dot{\varphi}Sin\varphi \end{Bmatrix} + \begin{Bmatrix} \text{-}q\dot{\alpha}Sin\alpha \\ \text{-}q\dot{\beta}Sin\beta \\ \text{-}q\dot{\gamma}Sin\gamma \end{Bmatrix} + \begin{Bmatrix} \text{-}r\dot{\theta}Sin\theta \\ r\dot{\theta}Cos\theta \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{8}$$

and

$$\dot{\alpha}Sin2\alpha+\dot{\beta}Sin2\beta+\dot{\gamma}Sin2\gamma=0 \tag{9}$$

Solving above equations simultaneously, we get

$$\dot{\varphi}=\frac{\dot{\theta}r[Sin\theta Cos\alpha\text{-}Cos\theta Cos\beta]}{p[Cos\varphi Cos\alpha\text{-}Sin\varphi Cos\gamma]}$$

$$\dot{\alpha}=\frac{\text{-}r\dot{\theta}Sin\theta+p\dot{\varphi}Cos\varphi}{qSin\alpha}, \dot{\beta}=\frac{r\dot{\theta}Cos\theta}{qSin\beta}$$

$$\dot{\gamma}=\frac{\text{-}p\dot{\varphi}Sin\varphi}{qSin\gamma}$$

The output angular velocity ($\dot{\varphi}$) is dependent on the input velocity, link lengths and the different angles. Therefore, it will vary with time as α, β, γ, φ, and θ vary with time.

4.3 Acceleration Analysis

As output velocity varies with time, there is some angular acceleration associated with it. Also, for the dynamic analysis of the mechanism, analysis of the output acceleration is important. Differentiating Eqs. (8) and (9) with respect to time, i.e.,

$$\begin{Bmatrix} p\ddot{\varphi}Cos\varphi\text{-}p\dot{\varphi}^2Sin\varphi \\ 0 \\ \text{-}p\ddot{\varphi}Sin\varphi\text{-}p\dot{\varphi}^2Cos\varphi \end{Bmatrix} + \begin{Bmatrix} \text{-}q\ddot{\alpha}Sin\alpha\text{-}q\dot{\alpha}^2Cos\alpha \\ \text{-}q\ddot{\beta}Sin\beta\text{-}q\dot{\beta}^2Cos\beta \\ \text{-}q\ddot{\gamma}Sin\gamma\text{-}q\dot{\gamma}^2Cos\gamma \end{Bmatrix}$$

$$+ \begin{Bmatrix} \text{-}r\ddot{\theta}Sin\theta\text{-}r\dot{\theta}^2Cos\theta \\ r\ddot{\theta}Cos\theta - r\dot{\theta}^2Sin\theta \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{10}$$

and

$$\ddot{\alpha}Sin2\alpha+2\dot{\alpha}^2Cos2\alpha+\ddot{\beta}Sin2\beta+2\dot{\beta}^2Cos2\beta$$

$$+\ddot{\gamma}Sin2\gamma+2\dot{\gamma}^2Cos2\gamma=0 \tag{11}$$

Solving above two sets of equations simultaneously and using the fact that the input acceleration ($\ddot{\theta}$) is zero as the crank rotates with constant speed, we get output angular acceleration as

$$\ddot{\varphi}=\frac{d\dot{\varphi}^2+e\dot{\alpha}^2+f\dot{\beta}^2+g\dot{\gamma}^2+h\dot{\theta}^2}{pCos\alpha Cos\varphi\text{-}pCos\gamma Sin\varphi}$$

where,

d=p[SinφCosα+CosφCosγ],

e=qCos^{2}α-Cos2α, f=qCos^{2}β-Cos2β,

g=qCos^{2}γ-Cos2γ, and h=r[CosθCosα+SinθCosβ]

It is noted that output acceleration depends upon links dimensions, different angles, and associated angular velocities at a particular instant.

5 Results

To see the variation of the output angular displacement (φ), angular velocity ($\dot{\varphi}$) and angular acceleration ($\ddot{\varphi}$) with crank rotation (θ), the relations developed in kinematic analysis are coded in Matlab. Link lengths are taken from the dimensional synthesis, i.e., r = 8 mm, x = 60 mm, p = 40 mm, and magnitude of the vector **q** as, q = r^2 + p^2 + x^2. From Fig. 19, the range of output angular displacement is about 15^0. The variation of these quantities and the range of the output angular displacement are in conformance with the results of the existing machine [2].

A real prototype is fabricated, as shown in Fig. 21, which is undergoing testing now.
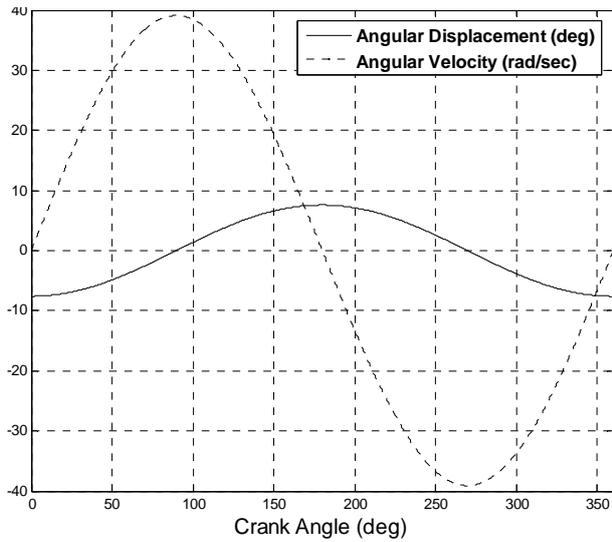
Figure 19: Output Angular Displacement and Velocity variation against Input Crank Angle
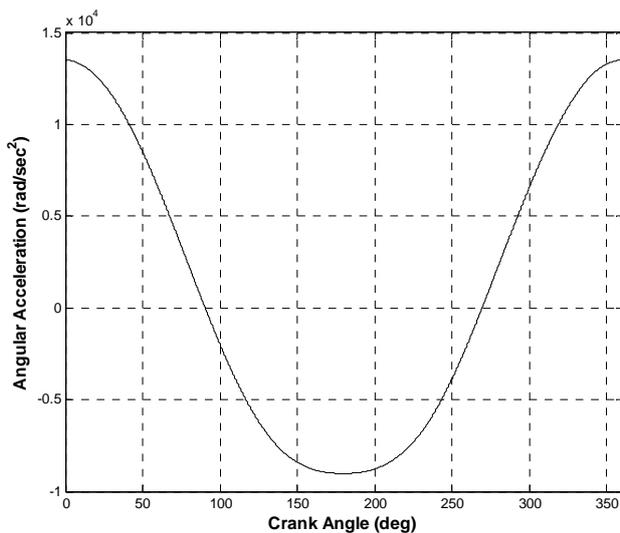
Figure 20: Output Angular Acceleration Variation

Figure 21: Prototype of the New Mechanism

6   Conclusions

Synthesis of a new mechanism to replace the existing mechanism of the sheep shearing machine is achieved using a systematic approach of mechanism selection and synthesis. It is expected that the new machine will have improved performance in terms of the wear and tear of the parts, as the new mechanism has no translational joint unlike in the existing mechanism [2]. Besides, the price will be cheaper, as it is indigenously developed, and any further improvement can be taken up in India itself.

Acknowledgement

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