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Dynamic Modelling of Serial-Link Mechanisms

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Abstract

The dynamic equations governing the motion of open-loop serial link mechanisms, which are also referred to as manipulators, are derived in this paper. The method is based on the Newton-Euler equations of motion of rigid bodies and Decoupled Natural Orthogonal Complement (DeNOC) matrix, introduced elsewhere. The DeNOC relates the angular and linear velocities of the rigid bodies in the system to the associated joint rates. When its transpose is multiplied to the uncoupled NE equations of rigid bodies, a set of constrained dynamic equations are obtained. The method is similar to some approaches in multibody dynamics, however, the novelty lies in the derivation of the DeNOC matrix. Moreover, the resulting equations permit recursive calculations for the inverse and forward dynamics, making control and simulation codes of mechanical system easier and computationally cheaper.

1 INTRODUCTION

The need for dynamic modelling of a mechanism or a manipulator, arises for its control and simulation. The common approach is to obtain the dynamic model of the system, consisting of rigid bodies coupled by kinematic pairs or joints, using the Newton-Euler(NE) or Euler-Lagrange(EL) equations. While NE equations are obtained from free-body diagram, the EL equations result from the kinetic and potential energy of the system. The former is not suitable for motion simulation, as it requires the elimination of the internal forces and torques that do not affect the motion of the system. Alternatively, EL equations give independent set of equations that are good for motion simulation but require complex partial derivatives. Note, however, by reformulating the problem those partial derivatives need not be evaluated explicitly (Silver, 1982). In the present methodology, one starts with NE equations and ends up with the EL equations using an orthogonal complement matrix (Angeles and Lee, 1988). This approach is well-known in multibody dynamics (Kamman and Huston, 1984). However, here the derivation of the orthogonal complement matrix, namely, the Decoupled Natural Orthogonal Complement(DeNOC) is different. The use of DeNOC, an orthogonal complement matrix that is naturally obtained from kinematic constraints and expressed as a product of two matrices, has the following features:

1. Analytical expressions for the elements of the matrices associated with the equations of motion of the mechanisms.
2. Physical interpretations of the elements of the matrices mentioned in item 1 above can be given.
3. Recursive forward (Saha, 1997) and inverse dynamics (Saha, 1995b) algorithms are possible.

Note that recursive forward dynamics algorithm is not possible, e.g., in Luh, Walker and Paul (1980).

2 DEFINITION OF THE DeNOC

For an n-degrees of freedom (DOF) open-loop serial-link mechanism (manipulator), as shown in Fig. 1, the n-dimensional joint rate vector, $\dot{\boldsymbol{\theta}}$, is defined as

$$\dot{\boldsymbol{\theta}} \equiv [\dot{\theta}_1, \cdots, \dot{\theta}_n]^T \qquad (1)$$

The twist of the ith rigid link, Fig. 1, undergo-

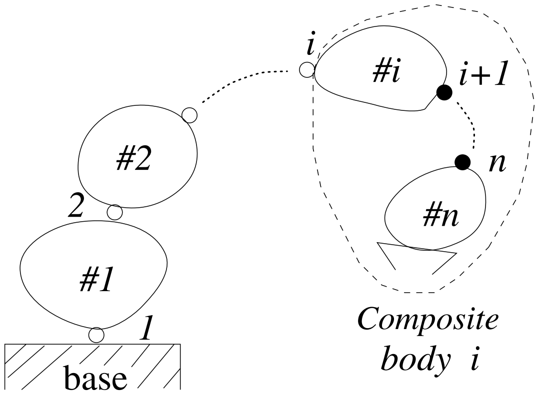

Figure 1: An n-link n-DOF serial manipulator.

ing motion in the three dimensional Cartesian space, $\mathbf{t}_i$, is defined in terms of its angular velocity, $\boldsymbol{\omega}_i$, and the velocity of the corresponding mass center, C_i, $\dot{\mathbf{c}}_i$. Hence $\mathbf{t}_i$ is the following six dimensional vector:

$$\mathbf{t}_i \equiv \begin{bmatrix} \boldsymbol{\omega}_i \\ \dot{\mathbf{c}}_i \end{bmatrix} \qquad (2)$$

Moreover, a $6n$-dimensional vector of generalized twist, $\mathbf{t}$, is defined as follows:

$$\mathbf{t} \equiv \begin{bmatrix} \mathbf{t}_1^T & \mathbf{t}_2^T & \cdots & \mathbf{t}_n^T \end{bmatrix}^T \qquad (3)$$

Now, from the kinematic constraints that exist between two successive links due to the joints, namely, link i and j of Fig. 2, $\mathbf{t}_i$ can be expressed as

$$\mathbf{t}_i = \mathbf{B}_{ij}\mathbf{t}_j + \mathbf{p}_i\dot{\theta}_i \qquad (4)$$

where the 6×6 matrix, $\mathbf{B}_{ij}$, and the 6-dimensional vector, $\mathbf{p}_i$, are given by

$$\mathbf{B}_{ij} \equiv \begin{bmatrix} \mathbf{1} & \mathbf{O} \\ \mathbf{C}_{ij} & \mathbf{1} \end{bmatrix}; \qquad \mathbf{p}_i \equiv \begin{bmatrix} \mathbf{e}_i \\ \mathbf{e}_i \times \mathbf{d}_i \end{bmatrix} \qquad (5)$$

in which $\mathbf{C}_{ij}$ is the 3×3 cross-product tensor associated with the vector, $\mathbf{c}_{ij} \equiv \mathbf{c}_j - \mathbf{c}_i$, as indicated in Fig. 2, such that for an arbitrary 3-dimensional vector, $\mathbf{x}$, $\mathbf{C}_{ij}\mathbf{x} \equiv \mathbf{c}_{ij} \times \mathbf{x}$. The vector $\mathbf{d}_i$ in the expression of $\mathbf{p}_i$ is shown in Fig. 2. Equation 5 is now written for all successive links of the n-link system, Fig. 1. The resultant kinematic constraint equation is

$$\mathbf{t} = \mathbf{T}\dot{\boldsymbol{\theta}} \qquad \text{where} \qquad \mathbf{T} \equiv \mathbf{T}_l\mathbf{T}_d \qquad (6)$$

$\mathbf{T}$ being the $6n \times n$ Natural Orthogonal Complement (NOC) matrix (Angeles and Lee, 1988), whereas the $6n \times 6n$ and $6n \times n$ Decoupled NOC (DeNOC) matrices, $\mathbf{T}_l$ and $\mathbf{T}_d$, respectively, are displayed below:

$$\mathbf{T}_l \equiv \begin{bmatrix} \mathbf{1} & \mathbf{O} & \cdots & \mathbf{O} \\ \mathbf{B}_{21} & \mathbf{1} & \cdots & \mathbf{O} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{B}_{n1} & \mathbf{B}_{n2} & \cdots & \mathbf{1} \end{bmatrix} \qquad (7)$$

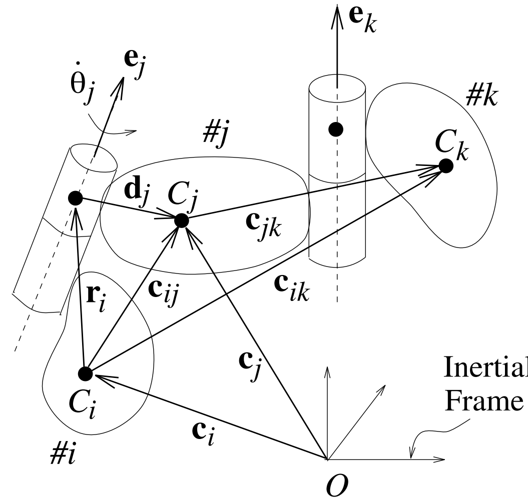

Figure 2: A system of coupled links.

and

$$\mathbf{T}_d \equiv \begin{bmatrix} \mathbf{p}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{p}_n \end{bmatrix} \qquad (8)$$

The 3×3 matrix, $\mathbf{1}$ is the identity matrix, and $\mathbf{O}$ and $\mathbf{0}$ are the matrix and vector of zeros, respectively. Note that the dimensions of $\mathbf{1}$, $\mathbf{O}$ and $\mathbf{0}$ are compatible to the dimensions where they appear. Thus $\mathbf{O}$ in $\mathbf{T}_l$ of eq.(7) is 6×6, whereas in eq.(5) it is 3×3. Moreover, the expressions, $\mathbf{B}_{ij}$, and $\mathbf{p}_i$, as appearing in eq.(5), have the following interpretations:

1. If the links $\#i$ and $\#j$, are rigidly attached, $\mathbf{B}_{ij}$, propagates the twist of $\#i$ to $\#j$.
2. Alternatively, vector, $\mathbf{p}_i$, takes into account the motion of ith revolute joint.

3 DYNAMIC MODELLING USING THE DeNOC

The system under study is shown in Fig.1. It has n links coupled by n joints. If $\mathbf{I}_i$ denotes the 3×3 moment of inertia tensor of the ith link about its mass center, C_i, and this, as well all as other vector quantities involved, are referred to a coordinate system fixed to the link, then Newton-Euler equations (NE) governing the motion of the ith link are written as follows (Angeles and Lee, 1988):

$$\mathbf{M}_i\dot{\mathbf{t}}_i + \mathbf{W}_i\mathbf{M}_i\mathbf{t}_i = \mathbf{w}_i \qquad (9)$$

where the 6×6 matrices $\mathbf{M}_i$ and $\mathbf{W}_i$, and the 6-dimensional wrench vector, $\mathbf{w}_i$, acting on ith link are defined as:

$$\mathbf{M_i} \equiv \begin{bmatrix} \mathbf{I}_i & \mathbf{O} \\ \mathbf{O} & m_i\mathbf{1} \end{bmatrix}; \quad \mathbf{W}_i \equiv \begin{bmatrix} \mathbf{\Omega}_i & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{bmatrix}; \quad \mathbf{w}_i \equiv \begin{bmatrix} \mathbf{n}_i \\ \mathbf{f}_i \end{bmatrix}$$

in which $\mathbf{\Omega}_i$ is the 3×3 cross-product tensor associated to the angular velocity, $\boldsymbol{\omega}_i$, that is defined similar to $\mathbf{C}_{ij}$ of eq.(5). Moreover, $\mathbf{n}_i$ and $\mathbf{f}_i$ are the 3-dimensional vectors, the former denoting the resultant moment about C_i, and the later the resultant forces acting at C_i. When written for all n links, uncoupled NE equations, i.e., eq.(9) for $i = 1, \cdots, n$, can be expressed in compact form as

$$\mathbf{M}\dot{\mathbf{t}} + \mathbf{WMt} = \mathbf{w} \qquad (10)$$

Here $6n \times 6n$ generalized mass matrix, $\mathbf{M}$, the $6n \times 6n$ generalized matrix of angular velocities, $\mathbf{W}$, and the $6n$-dimensional generalized vector of wrench, $\mathbf{w}$, are as follows:

$$\mathbf{M} \equiv \text{diag}(\mathbf{M}_1, \cdots, \mathbf{M}_n); \quad \mathbf{W} \equiv \text{diag}(\mathbf{W}_1, \cdots, \mathbf{W}_n); \quad \mathbf{w} \equiv [\mathbf{w}_1^T, \cdots, \mathbf{w}_n^T]^T$$

premultiplying eq.(10) by $\mathbf{T}^T$ we get n independent dynamic equations of motion of the coupled system (Angeles and Lee, 1988) shown in Fig. 1 as

$$\mathbf{T}^T(\mathbf{M}\dot{\mathbf{t}} + \mathbf{WMt}) = \mathbf{T}^T(\mathbf{w}^E + \mathbf{w}^C) \qquad (11)$$

where $\mathbf{w}$ is substituted by $\mathbf{w} \equiv \mathbf{w}^C + \mathbf{w}^E$, $\mathbf{w}^C$ and $\mathbf{w}^E$ being the $6n$-dimensional vectors of generalized constraint and external wrenches, respectively. Note that $\mathbf{T}^T\mathbf{w}^C$ vanishes, as constraint wrench produces no work. Moreover, upon substitution of eq.(6) and its time derivative in eq.(11), we get n independent scalar dynamical equations of motion, i.e.,

$$\mathbf{I}\ddot{\boldsymbol{\theta}} + \mathbf{C}\dot{\boldsymbol{\theta}} = \boldsymbol{\tau} \qquad (12)$$

where
$\mathbf{I} \equiv \mathbf{T}^T\mathbf{MT}$: the $n \times n$ generalized inertia matrix;
$\mathbf{C} \equiv \mathbf{T}^T(\mathbf{M}\dot{\mathbf{T}} + \mathbf{WMT})$: the $n \times n$ matrix of convective inertia terms;
$\boldsymbol{\tau} \equiv \mathbf{T}^T\mathbf{w}^E$: the n-dimensional vector of generalized forces due to driving forces/torques and those resulting from gravity and dissipation.

3.1 Symbolic Representation of I and C

Using DeNOC, the symbolic representation of matrices, $\mathbf{I}$ (GIM) and $\mathbf{C}$ are derived in the following sections.

3.1.1 Derivation of GIM

Substituting the expression of the DeNOC, eq.(**??**), into that of $\mathbf{I}$ of eq.(12), the GIM can be written as

$$\mathbf{I} \equiv \mathbf{T}_d^T\tilde{\mathbf{M}}\mathbf{T}_d \quad \text{where} \quad \tilde{\mathbf{M}} \equiv \mathbf{T}_l^T\mathbf{MT}_d \qquad (13)$$

in which, the $6n \times 6n$ symmetric matrix, $\tilde{\mathbf{M}}$, can be written by substituting eq.(**??**) into eq.(13) as

$$\tilde{\mathbf{M}} \equiv \begin{bmatrix} \tilde{\mathbf{M}}_1 & \mathbf{B}_{21}^T\tilde{\mathbf{M}}_2 & \cdots & \mathbf{B}_{n1}^T\tilde{\mathbf{M}}_n \\ \tilde{\mathbf{M}}_2\mathbf{B}_{21} & \tilde{\mathbf{M}}_2 & \cdots & \mathbf{B}_{n2}^T\tilde{\mathbf{M}}_n \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\mathbf{M}}_n\mathbf{B}_{n1} & \tilde{\mathbf{M}}_n\mathbf{B}_{n2} & \cdots & \tilde{\mathbf{M}}_n \end{bmatrix} \qquad (14)$$

where 6×6 matrix, $\tilde{\mathbf{M}}_i$, for $i = 1, \cdots, n$, can be calculated as

$$\tilde{\mathbf{M}}_i = \mathbf{M}_i + \mathbf{B}_{i+1,i}^T\tilde{\mathbf{M}}_{i+1}\mathbf{B}_{i+1,i} \qquad (15)$$

where $\tilde{\mathbf{M}}_{n+1} \equiv \mathbf{O}$ because there is no $(n+1)$st body in the system. Thus, matrix $\tilde{\mathbf{M}}_i$ is the mass matrix ofthe composite body, i, that consists of rigidly connected bodies, $\#i, \cdots, \#n$, as indicated in Fig. **??**. The GIM, $\mathbf{I}$ of eq.(12) is now expressed as

$$\mathbf{I} \equiv \begin{bmatrix} i_{11} & & \text{sym} \\ \vdots & \ddots & \\ i_{n1} & \cdots & i_{nn} \end{bmatrix}, \text{ where} i_{ij} \equiv \mathbf{p}_i^T\tilde{\mathbf{M}}_i\mathbf{B}_{ij}\mathbf{p}_j \qquad (16)$$

for $i = 1, \cdots, n$; $j = 1, \cdots, i$. Also, i_{ij} is scalar and "sym" denotes the symmetric elements of matrix $\mathbf{I}$.

3.1.2 Derivation of C matrix

Using the DeNOC, eq.(**??**) and its time derivative, the matrix, $\mathbf{C}$, as defined after eq.(12), is given**??**) by

$$\mathbf{C} \equiv \mathbf{T}_d^T\tilde{\mathbf{M}}'\mathbf{T}_d, \text{where } \tilde{\mathbf{M}}' \equiv \tilde{\mathbf{M}}\mathbf{V} + \mathbf{T}_l^T\mathbf{M}\dot{\mathbf{T}}_l + \dot{\tilde{\mathbf{M}}} \qquad (17)$$

in which the following identities are used:

$$\dot{\mathbf{T}} = \mathbf{VT}_d, \text{ and } \dot{\tilde{\mathbf{M}}} = \mathbf{T}_l^T\dot{\mathbf{M}}\mathbf{T}_l \qquad (18)$$

$\mathbf{V}$ being the $6n \times 6n$ generalized angular velocity matrix of the coordinate frames, which is defined as

$$\mathbf{V} \equiv \text{diag}(\mathbf{V}_1, \cdots, \mathbf{V}_n); \ \mathbf{V}_i \equiv \text{diag}(\boldsymbol{\omega}_i \times \mathbf{1}, \boldsymbol{\omega}_i \times \mathbf{1}) \qquad (19)$$

The elements of the $n \times n$ matrix, $\mathbf{C}$, c_{ij}, for $i, j = 1, \cdots, n$ are expressed**??**) as

$$c_{ij} = \mathbf{p}_i^T(\mathbf{B}_{ji}^T\tilde{\mathbf{M}}_j\mathbf{V}_j + \mathbf{B}_{j+1,i}^T\tilde{\mathbf{H}}_{j+1} + \dot{\tilde{\mathbf{M}}}_j)\mathbf{p}_j \quad \text{if} \quad i \le j$$
$$c_{ij} = \mathbf{p}_i^T(\tilde{\mathbf{M}}_i\mathbf{B}_{ij}\mathbf{V}_j + \tilde{\mathbf{H}}_{ij} + \dot{\tilde{\mathbf{M}}}_i)\mathbf{p}_j \quad \text{otherwise} \qquad (20)$$

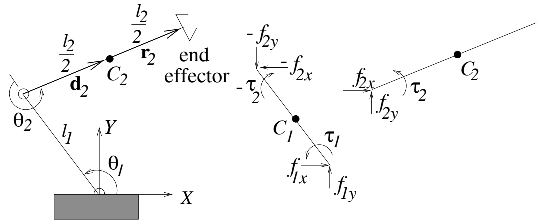

Figure 3: A 2-DOF planar mechanisms.

where

$$\tilde{\mathbf{H}}_i \equiv \tilde{\mathbf{H}}_{i,i-1}; \quad \tilde{\mathbf{H}}_{ij} \equiv \tilde{\mathbf{M}}_i \dot{\mathbf{B}}_{ij} + \mathbf{B}^T_{i+1,i} \tilde{\mathbf{H}}_{i+1} \tag{21}$$

in which $\tilde{\mathbf{H}}_{n+1} \equiv \mathbf{O}$. Moreover, the 6×6 matrix, $\dot{\tilde{\mathbf{M}}}_i$, as in above equation, for $i = 1, \cdots, n$ are given as, $\dot{\tilde{\mathbf{M}}}_i = \mathbf{W}_i \mathbf{M}_i + \dot{\tilde{\mathbf{M}}}_{i+1}$, where $\tilde{\mathbf{M}}_{n+1} = \mathbf{O}$.

4 EXAMPLE: A 2-DOF MANIPULATOR

The dynamic modelling technique outlined above is illustrated with a two degrees of freedom (DOF) serial-link manipulator, as shown in Fig. 3. The system has two revolute joints. The link lengths are l_1 and l_2. The elements of the associated 2×2 matrices, $\mathbf{I}$ and $\mathbf{C}$, as derived in sections, 3.1.1, and 3.1.2, respectively, are shown below:

$$\begin{aligned} i_{11} &= \mathbf{p}_1^T \tilde{\mathbf{M}}_1 \mathbf{B}_{11} \mathbf{p}_1 = I_1 + I_2 + \frac{1}{4}(m_1 l_1^2 \\ &\quad + m_2 l_2^2) + m_2 l_1^2 + m_2 l_1 l_2 \cos\theta_2 \\ &= \frac{1}{3}(m_1 l_1^2 + m_2 l_2^2) + m_2 l_1^2 + m_2 l_1 l_2 \cos\theta_2 \\ i_{12} &= i_{21} = \mathbf{p}_2^T \tilde{\mathbf{M}}_2 \mathbf{B}_{21} \mathbf{p}_1 = I_2 + \frac{m_2 l_2^2}{4} \\ &\quad + m_2 l_1 l_2 \cos\theta_2 \\ &= \frac{1}{3} m_2 l_2^2 + \frac{1}{2} m_2 l_1 l_2 \cos\theta_2 \\ i_{22} &= \mathbf{p}_2 T \tilde{\mathbf{M}}_2 \mathbf{B}_{22} \mathbf{p}_2 = I_2 + \frac{m_2 l_2^2}{4} = \frac{1}{3} m_2 l_2^2 \end{aligned}$$

where $I_1 \equiv \frac{1}{12} m_1 l_1^2$ and $I_2 \equiv \frac{1}{12} m_2 l_2^2$ are the moment of inertia of the 1st and 2nd links about the axis normal to the plane of motion, i.e., $X - Y$ plane, and passing through C_1 and C_2, respectively. Note above that $\mathbf{B}_{11} = \mathbf{B}_{22} = \mathbf{1}$, $\tilde{\mathbf{M}}_1 \equiv \mathbf{M}_1 + \mathbf{B}_{21}^T \tilde{\mathbf{M}}_2 \mathbf{B}_{21}$, and $\tilde{\mathbf{M}}_2 = \mathbf{M}_2$. Also,

$$\begin{aligned} \mathbf{p}_1 &\equiv [0,\ 0,\ 1,\ -\frac{1}{2} l_1 \sin\theta_1,\ \frac{1}{2} l_1 \cos\theta_1,\ 0]^T \\ \mathbf{p}_2 &\equiv [0,\ 0,\ 1,\ -\frac{1}{2} l_2 \sin\theta_{12},\ \frac{1}{2} l_2 \cos\theta_{12},\ 0]^T \end{aligned}$$

and

$$\mathbf{B}_{21} = \begin{bmatrix} \mathbf{1} & \mathbf{O} \\ \mathbf{C}_{21} & \mathbf{1} \end{bmatrix}$$

in which $\mathbf{C}_{21}$ is the 3×3 cross-product tensor associated to the vector, $\mathbf{c}_{21}$, defined similar to $\mathbf{C}_{ij}$ of eq.(5). The vector $\mathbf{c}_{21}$ is given by

$$\mathbf{c}_{21} \equiv -\frac{1}{2}[(l_2 \cos\theta_{12} + l_1 \cos\theta_1,\ l_2 \sin\theta_{12} + l_1 \sin\theta_1,\ 0]^T$$

and $\theta_{12} \equiv \theta_1 + \theta_2$.

The elements of matrix, $\mathbf{C}$, for the manipulator, Fig. 3, are as follows:

$$\begin{aligned} c_{11} &= \mathbf{p}_1^T (\mathbf{B}_{11}^T \tilde{\mathbf{M}}_1 \mathbf{V}_1 + \mathbf{B}_{21}^T \tilde{\mathbf{H}}_2 + \dot{\tilde{\mathbf{M}}}_1) \mathbf{p}_1 \\ &= -\frac{1}{2} m_2 l_1 l_2 \sin\theta_2 \dot{\theta}_2 \\ c_{12} &= \mathbf{p}_1^T (\mathbf{B}_{21}^T \tilde{\mathbf{M}}_2 \mathbf{V}_2 + \mathbf{B}_{31}^T \tilde{\mathbf{H}}_3 + \dot{\tilde{\mathbf{M}}}_2) \mathbf{p}_2 \\ &= -\frac{1}{2} m_2 l_1 l_2 \sin\theta_2 (\dot{\theta}_1 + \dot{\theta}_2) \\ c_{21} &= \mathbf{p}_2^T (\tilde{\mathbf{M}}_2 \mathbf{B}_{21} \mathbf{V}_2 + \tilde{\mathbf{H}}_{21} + \dot{\tilde{\mathbf{M}}}_1) \mathbf{p}_1 \\ &= \frac{1}{2} m_2 l_1 l_2 \sin\theta_2 \dot{\theta}_1 \\ c_{22} &= \mathbf{p}_2 T (\mathbf{B}_{22}^T \tilde{\mathbf{M}}_2 \mathbf{V}_2 + \mathbf{B}_{32} T \tilde{\mathbf{H}}_3 + \dot{\tilde{\mathbf{M}}}_3) \mathbf{p}_2 = 0 \end{aligned}$$

where $\dot{\tilde{\mathbf{M}}}_2 = \dot{\tilde{\mathbf{M}}}_1 = \dot{\tilde{\mathbf{H}}}_3 = \mathbf{O}$ and

$$\dot{\mathbf{B}}_{21} \equiv \begin{bmatrix} \mathbf{O} & \mathbf{O} \\ \dot{\mathbf{C}}_{21} & \mathbf{O} \end{bmatrix}$$

in which $\dot{\mathbf{C}}_{21}$ is the 3×3 cross-product tensor associated to the vector $\dot{\mathbf{c}}_{21}$ that is given by $\dot{\mathbf{c}}_{21} \equiv \frac{1}{2}[l_2 \sin\theta_{12}(\dot{\theta}_1 + \dot{\theta}_2) + l_1 \sin\theta_1 \dot{\theta}_1,\ l_2 \cos\theta_{12}(\dot{\theta}_1 + \dot{\theta}_2) + l_1 \cos\theta_1 \dot{\theta}_1,\ 0]^T$

5 CONCLUSIONS

A systematic development of dynamic equations of motion of open-loop serial-link mechanisms, namely, manipulators, is presented in this paper. This method is illustrated with a 2-DOF planar manipulator. The results are verified with the EL equations of motion. The dynamic modelling of a 3-DOF planar manipulator is also performed and verified with EL equations, but not included here for their complex expressions, particularly, for i_{11} and c_{11}. The methodology, however, is same as illustrated for the 2-DOF system.

The dynamic modelling using the DeNOC has the following features:

- Symbolic representation of the elements of the matrices of $\mathbf{I}$ and $\mathbf{C}$, given in eqs.(16) and (20), respectively.
- Physical interpretations of many terms, e.g., as explained in Section 2, can be given.
- Recursive algorithms of order n for inverse and forward dynamics are possible. The latter is not possible with conventional algorithm, e.g., in Luh, Walker and Paul (1980).

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